Structural Optimization for Stabilized and Stiffened Structural System by Tension Members

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1. Abstract
This paper presents a numerical method for stabilized and stiffened structural system by tension members. In these structures, it is generally difficult to control a shape and prestress because of high dependency between them. Our proposal model has similar features of these systems, and we find the resisting structural form efficiently for compression occurred by prestressing.
Our analytical approach is to divide a structure into two groups which are instable system and statically indeterminate system. We formulated equations of stabilizing process in each group. These shapes of structures become a unique shape under the specified prestress. In previous research [1], we analyzed two-dimensional models and verified the proposed method. In this paper, we analyzed three-dimensional models and show some results which would be available to apply for spatial structures. Next, we present the optimization method for these structures. In this method, we define the strain energy as an objective function, and magnitude of prestress as design variables. It is difficult to apply linear analysis of stress and displacement in these structural models, because it would be instable without prestress. Therefore, we apply the geometrically nonlinear analysis with prestress by FEM. We focus a form-finding analysis and optimization for these structures in this paper.
2. Keywords: Structural optimization, Stabilized and Stiffened Structural System, Genetic Algorithm

3. Introduction
This paper presents a numerical method for a structural system stiffened by tension members, such as tensegrity structures and cable domes. These structures are utilized for a spatial structure with light weight and attractive appearance. However, in engineering process, it is generally difficult to control a shape and prestress. Therefore, we have to apply the form-finding analysis for an equilibrium shape. Many numerical analysis methods of such structures have been proposed and it would be successful methods. On the other hand, similar types of these structures have been investigated and constructed. These are called “tension-stabilizing truss”, and “tensegric dome” [2]. A different feature from tensegrity is that these systems are utilized for comparatively low stiffness structures, or instable structures. These systems are useful to stiffen the single-layered truss shell. We focus a form-finding analysis and optimization for these structures in this paper.

4. Isotonic Soap Films Stretched on the Polyhedron
Isotonic soap film stretched on the polyhedron is a conceptual model of our proposed structural one. J. Plateau, a Belgian physicist in the 19th Century, formulated Plateau’s law which describes the structure of bubble soap films from his experimental observation (Fig.1). It shows that isotonic soap films are stretched on the polyhedron wire frame. It also can be assumed that the equilibrium shape by self-stress exists in the member arrangement which is a state of tension members covered with compression members. That is to say, in the case of low stiffness shell, it can be stiffened by only tension members. Besides, this structural system is not necessary to be rigid at the compression member’s joint, it would become light weight and high stiffness structure with prestressing.

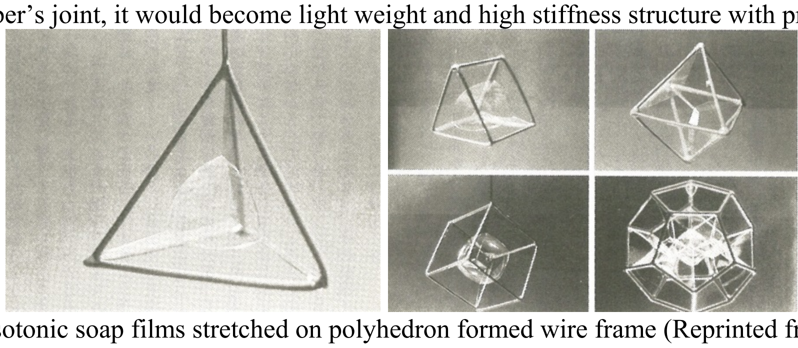

Fig.1 Isotonic soap films stretched on polyhedron formed wire frame (Reprinted from [3])

5. Numerical Analysis

5.1 Analytical Model

It is shown the initial shape consist of truss, cable members and pin joints in Fig.2. Variables of $p1$ and $p2$ are acting in the cable members. This model is a simple one of stabilized and stiffened structural system.

We explain the outline of form-finding analysis. Our approach is to divide a structure into two groups. The first (g1) is a group of instable system in which some rigid body displacements modes exist. This group consists of compression truss members shown in left side of Fig.4. Compression members could be rotated at the node, and bending moment doesn't occur inside of members. The second (g2) is a group of statically indeterminate system in which some self-equilibrating stress modes exist. This group consists of tension members shown in right side of Fig.4. In order to form these assumed systems, we must modify boundary conditions. To analyze these systems, we must set forcible deformations as initial values ($p1$, $p2$). And then, we control the shape of the structure by displacements toward the stable state. The g2 will be change the shape of the structure with specified deformations of members forcibly, and the g1 will be change the shape of the structure as rigid body displacement by acting reversed force of the g2's self-stress. In this way, we can obtain the stabilizing shape at the convergence. At last, forcible deformation as the initial values take the place of prestress values. The flow of analysis is shown in Fig.5.

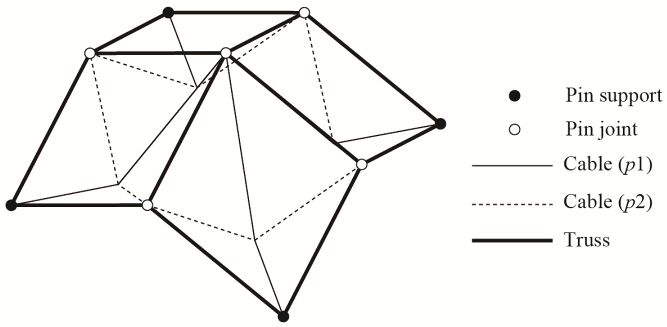

Fig.2 Initial shape and members

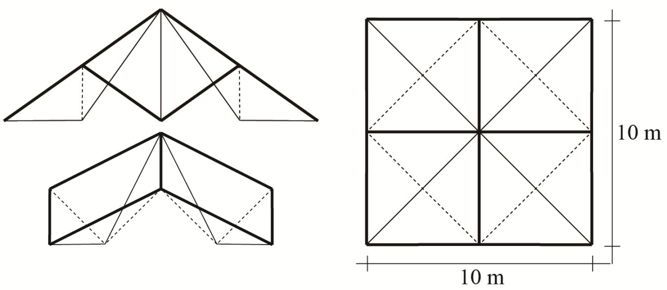

Fig.3 elevation (left side) and plan view (right side)

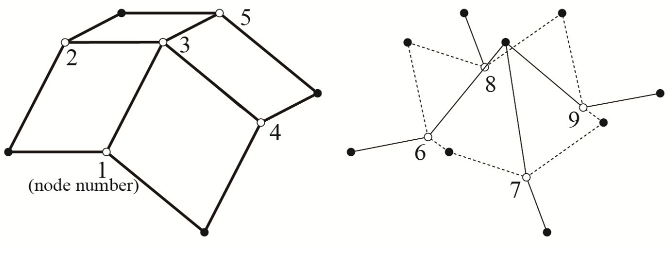

Fig.4 compression group (g1) and tension group (g2)

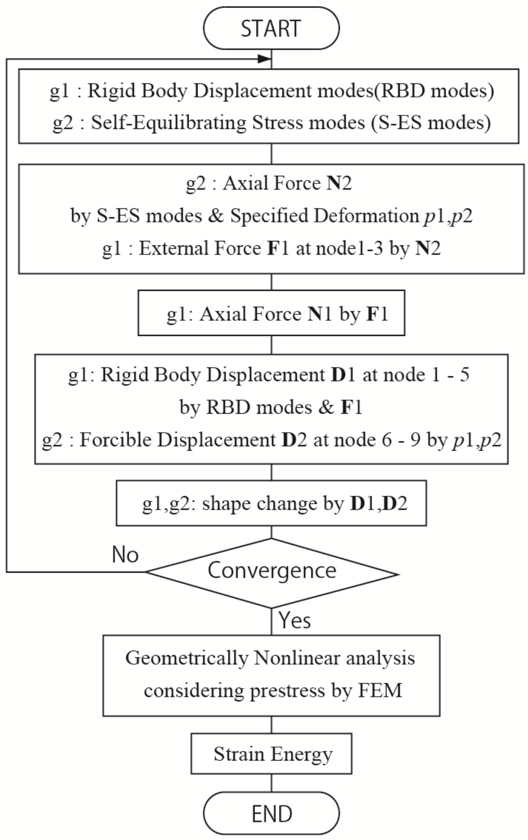

Fig.5 Analytical flow of form-finding

We investigate whether the structure has stiffness by the method of geometrically nonlinear analysis with prestress. It is assumed that the external force is acting as vertical load. The model could be applied linear analysis by FEM, but we have to avoid acting compression force into tension members. It appears that this stiffened system of structure would be useful.

5.2 Formulation of form-finding

The discretized static equilibrium equation of truss structures can be written as

$$\boldsymbol{An} = \boldsymbol{f} \tag{1}$$

A : equilibrium matrix, $\boldsymbol{n}$: axial force vector, $\boldsymbol{f}$: external force vector

The deformation–displacement relation can be written as

$$\boldsymbol{Bd} = \boldsymbol{\Delta l} \quad (2)$$

$\boldsymbol{B}$: deformation − displacement matrix, $\boldsymbol{d}$: displacement vector, $\boldsymbol{\Delta l}$: deformation vector

Now, we consider two groups of structural system. Rigid body displacement occurs by acting the External force in g1. Under the assumption of rigid body displacement, Eq. (2) becomes

$$\boldsymbol{Bd} = \boldsymbol{0} \text{ (in g1)} \quad (3)$$

$\boldsymbol{0}$: null vector

The Moore–Penrose generalized inverse matrix gives the displacement vector $\boldsymbol{d}$ as

$$\boldsymbol{d} = [\boldsymbol{I}_{\mathrm{n}} - \boldsymbol{B}^{+}\boldsymbol{B}]\boldsymbol{\alpha} \equiv \boldsymbol{H\alpha} \quad (4)$$

$\boldsymbol{I}_{\mathrm{n}}$: unit matrix(n, n), $\boldsymbol{B}^{+}$: Moore − Penrose generalized inverse matrix, $\boldsymbol{\alpha}$: arbitrary column vector

$\boldsymbol{H}$: matrix of rigid body displacement modes

To consider the convergence of the external potential energy, $\boldsymbol{d}$ can be written as

$$\boldsymbol{d} = \alpha \boldsymbol{Hf} \quad (5)$$

α : incrementation parameter

The axial force vector $\boldsymbol{n}$ in g1 can be written as

$$\boldsymbol{n} = -\left(\boldsymbol{B}^{+}\right)^{\mathrm{T}}\boldsymbol{f} \quad (6)$$

Obtained $\boldsymbol{n}$ in this way become least-square solutions.

Next, we formulate the displacement of g2. Acting the specified deformation causes the forcible displacement in g2. This specified deformation takes the place of prestress at the convergence of stabilizing process.

Under the assumption of self-equilibrium state, Eq. (1) becomes

$$\boldsymbol{An} = \boldsymbol{0} \text{ (in g2)} \quad (7)$$

The Moore–Penrose generalized inverse matrix gives the displacement vector $\boldsymbol{n}$ as

$$\boldsymbol{n} = [\boldsymbol{I}_{\mathrm{m}} - \boldsymbol{B}\boldsymbol{B}^{+}]\boldsymbol{\beta} \equiv \boldsymbol{G\beta} \quad (8)$$

$\boldsymbol{I}_{\mathrm{m}}$: unit matrix(m, m), $\boldsymbol{B}^{+}$: Moore − Penrose generalized inverse matrix, $\boldsymbol{\beta}$: arbitrary column vector

$\boldsymbol{G}$: matrix of self − equirbrating stress modes

To consider the convergence of the complementary energy, $\boldsymbol{n}$ can be written as

$$\boldsymbol{n} = -\boldsymbol{G\Delta l} \quad (9)$$

$\boldsymbol{\Delta l}$ is known value as specified deformation in g2. And, it is negative value. The external force of g1 can be obtained to calculate $\boldsymbol{n}$ of Eq. (9) such as Fig. 6.

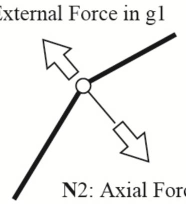

Fig.6 Relation of force in g1 and g2

The displacement of g2 can be written as

$$\boldsymbol{d} = \alpha \boldsymbol{B}^{+}\boldsymbol{\Delta l} \quad (10)$$

Obtained $\boldsymbol{d}$ become least-square solutions, and we regard $\boldsymbol{d}$ as the forcible displacement. To use $\boldsymbol{d}$ of Eq. (5), and (10), the shape of structure is changing. The convergence condition is in below.

$$e_1 = \boldsymbol{d}^{\mathrm{T}}\boldsymbol{f} \text{ (in g1)}$$
$$e_2 = (-\boldsymbol{\Delta l} - \boldsymbol{n})^{\mathrm{T}}(-\boldsymbol{\Delta l} - \boldsymbol{n}) \text{ (in g2)}$$
$$e = e_1 + e_2 < \varepsilon \quad (11)$$

$-\boldsymbol{\Delta l}$ becomes prestress when Eq. (11) is established. Therefore, the structure would be stabilized.

5.3 Geometrically Nonlinear Analysis with Prestress

In this paper, we show the geometrically nonlinear analysis with prestress by FEM. This analytical model stiffened by prestress, and it is important to include the geometric stiffness. This method is described as incremental analysis and updated Lagrange method. We apply the Newton-Raphson method as the solution of nonlinear equations.

In the incremental interval, equilibrium equation can be written as

$$\boldsymbol{K\Delta d} = \boldsymbol{\Delta f} \quad (12)$$

$\boldsymbol{K}$: tangent stiffeness matrix, $\boldsymbol{\Delta d}$: incremental displacement vector, $\boldsymbol{\Delta f}$: incremental force vector

Eq. (12) is expressed as below.

$$\boldsymbol{K} = \boldsymbol{K}_E + \boldsymbol{K}_G \quad (13)$$

$\boldsymbol{K}_E$: elastic stiffeness matrix, $\boldsymbol{K}_G$: geometric stiffness matrix

$$\boldsymbol{\Delta f} = \boldsymbol{r} = \boldsymbol{f} - \boldsymbol{q} \quad (14)$$

$\boldsymbol{r}$: residual force vector, $\boldsymbol{q}$: internal force vector

The prestress $\boldsymbol{n}_0$ is including in $\boldsymbol{K}_G$ and $\boldsymbol{q}$. $\boldsymbol{n}_0$ can be calculated by using Eq. (6), (8).

To determine deformation, axial force, and internal force, we use $\boldsymbol{B}$ matrix of the structure.

$$\boldsymbol{\Delta l} = \boldsymbol{B\Delta d} \quad (15)$$

$$n_k = \frac{E_k A_k}{l_k} \Delta l_k \tag{16}$$

$$\boldsymbol{q} = B^{\mathrm{T}}(\boldsymbol{n}_0 + \boldsymbol{n}) \tag{17}$$

E : elastic modulus, A : sectional area of member

n_k is the axial force in the process of deformation by external force acting. According to the Newton-Raphson method, $\boldsymbol{\Delta d}$ can be calculated as

$$\boldsymbol{\Delta d} = K^{-1}\boldsymbol{r} \tag{18}$$

To quantify the stiffness of the structure, we define the strain energy $\boldsymbol{U}$ as below. Considering the effect of prestress, it is expressed by using strain energy including prestress.

$$\boldsymbol{U} = \sum_{k=1}^{m} (n_{0k}\Delta l_{0k} + n_k \Delta l_k) \tag{19}$$

5.4 Result of Form-finding Analysis

We show the analysis for a fundamental type of structure (Fig.3). This model has 12 compression members and 16 tension members. These are truss members and cable members. We assume that cable members are symmetrically prestressing. Thus, we set a limit to the number of parameters, and we can draw a strain energy surface of various shape structures. Two results (A,B) of numerical analysis and physical property for FEM are shown as below.

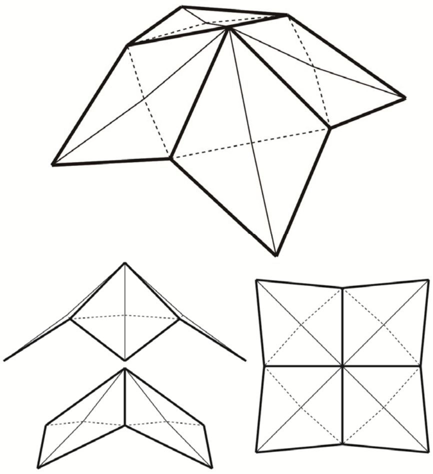

Fig.7 Analysis result-**A** ($p1$=1.80, $p2$=1.10)

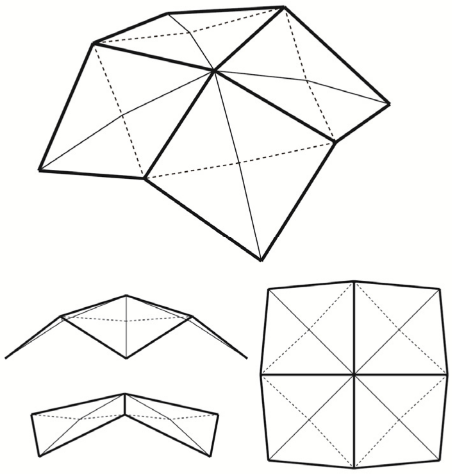

Fig.8 Analysis result-**B** ($p1$=1.15, $p2$=1.40)

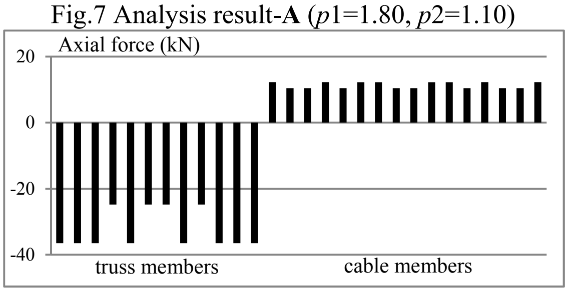

Fig.9 Axial force in result-**A**

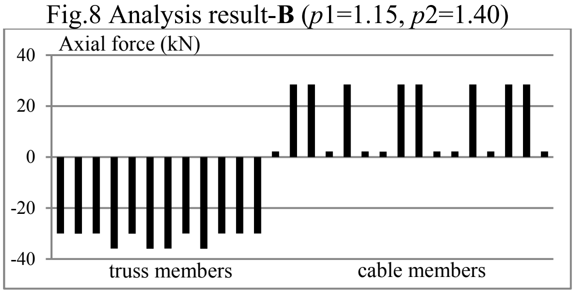

Fig.10 Axial force in result-**B**

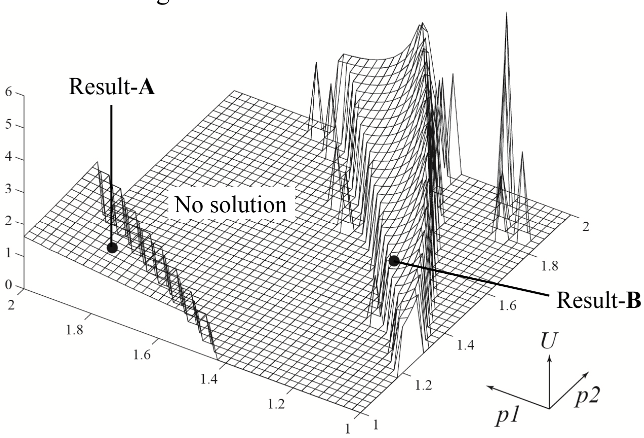

Fig.11 Strain energy surface

Tab.1 Physical property and analysis condition

Vertical load	1.0 kN/m^2
Young's modulus (truss)	2.1×10^8 kN/m^2
Young's modulus (cable)	1.4×10^8 kN/m^2
Section area (truss)	9.8×10^{-4} m^2
Section area (cable)	9.5×10^{-5} m^2
Coefficient for prestress	10

Form of analysis result of form-finding in section 5.2 is shown in Fig.7,8. Changing the variables $p1$, $p2$, various forms can be obtained. The physical property and analysis condition for geometrically nonlinear analysis with prestress are shown in Tab.1. The truss members are assumed the steel pipes of D = 101.6 mm. The cable members are assumed the strand cables of D = 14.0 mm. Coefficient for prestress is weight factor of prestress. $\boldsymbol{n}_0$ of Eq.(17) **i**nclude the coefficient for prestress in formulation.
Axial force in geometrically nonlinear analysis is shown in Fig.9, 10. It appears that compression members are acted negative axial force, and tension members are acted positive axial force. This result means that it is effective to appy prestress for this model. If this model have solved by using linear analysis in FEM, tension members would be acted negative axial force.
Strain energy surface in Fig. 11 show that this analysis is difficult to converge for optimization. Two cases of problem exist, the first is cable members cannot have almost same value of $p1$ and $p2$, it occurs vibration in form-finding. The second in FEM is the Coefficient for prestress is comparatively low value, tension members are acted negative axial force. Moreover, if the coefficient for prestress increase, compression force also increase in truss members. Thus, structural optimization is difficult to solve stably in this strain energy field.

6. Structural Optimization by Genetic Algorithm

In this paper, we apply Genetic Algorithm (GA) for structural optimization. GA is useful for an optimization problem such as finding global optimal solution without the gradient of objective function. In our proposed model, it is difficult to define the relationship between variables $p1$, $p2$ and strain energy, the gradient of energy cannot be obtained. Therefore, we chose GA for this problem. However, if we execute GA in this problem, many of individuals are fall in "No solution" region in Fig.11. It would not be efficient to converge. Therefore, we propose to change the variable space like Fig.12. This method is useful to avoid the region in which algorithm search. The formulation of optimization is shown in below.

$$\begin{array}{ll} \textit{Find} & p1, p2 \\ \textit{to minimize} & f(p1, p2) \\ \textit{subject to} & p^{L1} < p1 < p^{U1} \\ & p^{L2} < p2 < p^{U2} \end{array}$$

$f(p1, p2)$: Strain energy in Eq. (19)

$p^{L1}, p^{U1}, p^{L2}, p^{U2}$: Upper and lower constrain

$p^{L1} = 1.0,\ p^{U1} = 1.4,\ p^{L2} = 1.375\,p1 + 0.025,\ p^{U2} = 2.0$

These formulation search the hill in which the Result-**A** exist in Fig. 11. The parameter of GA is shown in Tab. 2. We executed the GA optimization three times. Thus, we show the result of optimization in Fig. 13-15. The relationship between strain energy and Iteration number is shown in Fig.13. The relationship between entropy Dp and Iteration number is shown in Fig.14. Entropy Dp is the value of assessment for convergence. It appears that these GA optimizations are in the state of convergence and finished in all trials.

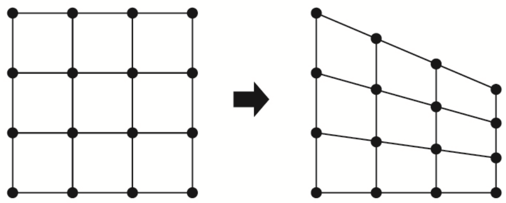

Fig.12 Mapping of variable space

Tab.2 Parameter of GA

Individual number	100	Generation number	50
Mutation ratio	0.06	selection	tournament elite
Crossover ratio	0.7	string	16bit

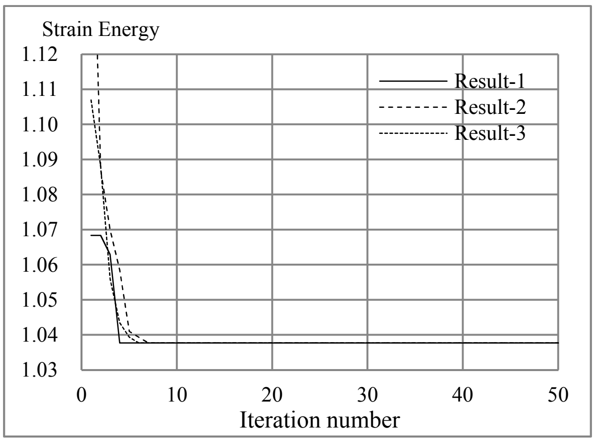

Fig.13 Strain energy – Iteration number

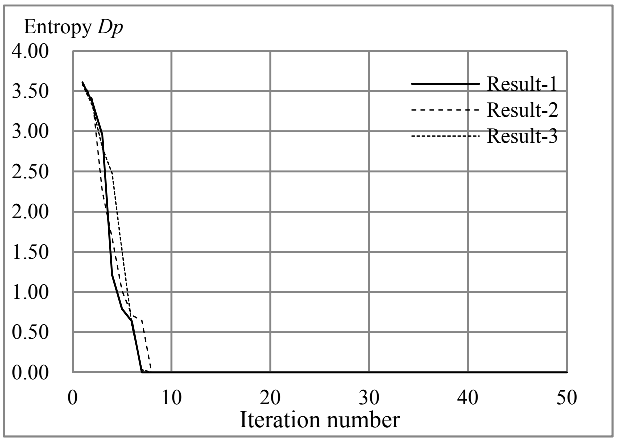

Fig.14 Entropy Dp – Iteration number

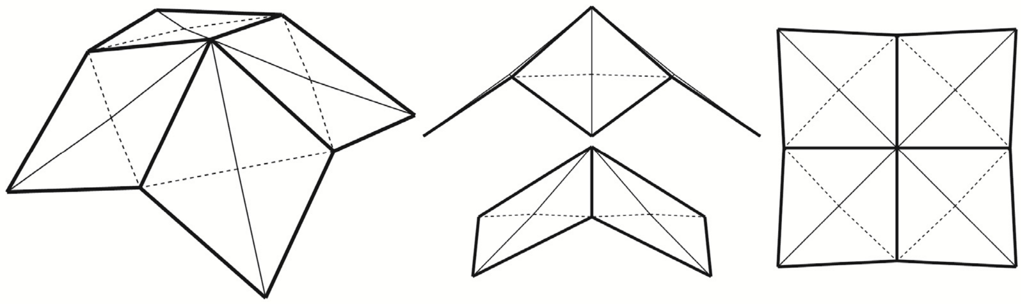

Fig.14 Form of GA optimization result ($p1$ = 1.433, $p2$ = 1.000, $f(p1, p2)$ = 1.038)

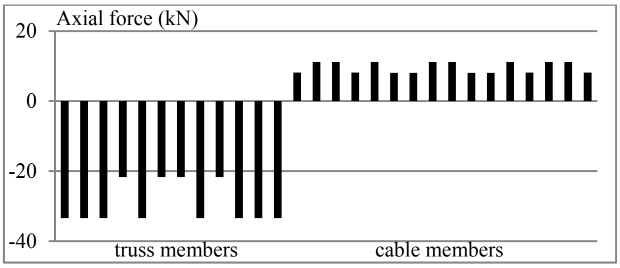

Fig.15 Axial force in GA optimization result

The form of GA optimization result is shown in Fig. 14. Axial force is shown in Fig. 15. Comparing with strain energy surface, the point of variables obtained by optimization is an optimal solution in the set region.
Although axial force tends to be in the state of result-**A** in form-finding, the values of axial force are totally low.

7. Conclusion

This paper presents a numerical method for a structural system stiffened by tension members. The form-finding analysis can obtain the form in equilibrium state with prestress. The geometrically nonlinear analysis allows to apply the structure with prestress. The strain energy surface show that this model is difficult to converge for optimization, but mapping of variable space is useful for the case including the "No solution" region. The structural optimization by GA is available to find optimal solution for a high stiffened structure.
This model is simple and static structure in the view of relation of member connection. This proposed method is useful for more complicated model such as instable structure with more tension members. The next step is to arrange these stabilized and stiffened structural models from every direction.

8. References

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