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Acoustic radiation and sensitivity analysis of a randomly excited structure based on FEM/IBEM combined with PEM

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Abstract
The objective of this study is to develop a computational method for calculating the acoustic radiation and sensitivity analysis of a structure subjected to a stochastic excitation, based on the finite element method (FEM), the indirect boundary element method (IBEM) and the pseudo excitation method (PEM). In this work, FEM and IBEM are used respectively to calculate the dynamic and acoustic responses of a structure, and PEM is used to determine the acoustic stochastic responses for the acoustic radiation problems via transforming the random responses into the structural-acoustic harmonic ones. Using the PEM, the acoustic radiation sensitivities of the structure are developed in the context of the transformed harmonic sensitivity analyses, and they are validated by comparing with the results predicted using the finite difference method. Numerical example is given to demonstrate the effectiveness of the methods proposed in this paper.
Keywords: Sensitivity analysis, Stochastic Excitation, FEM, IBEM, PEM

1. Introduction

The sensitivity analysis and design optimization have become an effective means of reducing vibration and noise in many areas of practical engineering in recent years. Wang and Lee [1] developed a global design sensitivity analysis of exterior noise with respect to structural sizing design variables. Allen et al. [2] presented a study on the stochastic acoustic radiation and sensitivity analysis. Liu et al. [3] proposed a new effective method for computing the acoustic radiation and its sensitivity analysis of a structure subject to a stochastic excitation.

The aim of the present work is to determine the acoustic power spectral density (PSD) and its sensitivity on a structural-acoustic system subjected to a stochastic excitation. FEM and IBEM are combined with an accurate and highly effective algorithm for stationary/non-stationary random structural response analysis, named as PEM, to solve the acoustic random radiation problem. PEM and FEM are used to calculate the pseudo responses of the structural vibration when the stochastic excitations are applied on the structure. IBEM is used to calculate the random acoustic radiation analysis, in which the structural pseudo response constitutes the boundary condition in acoustic indirect boundary element analysis. Thus, the acoustic PSD analysis could be obtained by means of harmonic analysis, and this method will make the calculation procedure of random acoustic analysis highly simple and efficient.

2. Structural random response analysis

A brief introduction of the structural random response analysis is given based on the PEM in this section [4], and that constitutes the boundary condition of the subsequent acoustic random response analysis. The finite element system equation for a structure subjected to a single random excitation can be expressed as follows:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=\{R_x\}x(t) \tag{1}$$

where $x(t)$ is a stationary random process with a specified PSD $S_{xx}(\omega)$ for which the transformation between them is not considered, and $\{R_x\}$ is a given constant vector represents the distribution of the random excitation. According to the PEM, substituting $\tilde{x}(t)=\sqrt{S_{xx}(\omega)}e^{i\omega t}$, as a pseudo excitation ($\tilde{\#}$ represents the pseudo variable of the random variable $\#$), into Eq.(1), it leads to the following traditional harmonic equation:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=\{R_x\}\sqrt{S_{xx}(\omega)}e^{i\omega t} \tag{2}$$

The solution of Eq.(2) can be easily obtained , such as $\{\tilde{y}(t)\}=\{Y(\omega)\}e^{i\omega t}$, by using mode-superposition method or other methods, the PSD matrix of $\{y\}$ can be computed as follows:

$$[S_{yy}(\omega)]=\{\tilde{y}\}^*\{\tilde{y}\}^{\mathrm{T}}=\{Y(\omega)\}^*\{Y(\omega)\}^{\mathrm{T}} \tag{3}$$

where the superscripts * and T represent the complex conjugate and transpose respectively, and a detailed description of the PEM for the structural random response analysis can be found in Ref. [4].

3. Acoustic random radiation analysis

Considering a structural velocity boundary condition for simplicity, the acoustic system equation in IBEM can be expressed as follows [5]:

$$[A]\{q\} = \{f\} \tag{4}$$

where $[A]$ is the acoustic system matrix, $\{q\}$ is the vector of unknown primary variables on the surface of the boundary element model, $\{f\}$ is the vector of the excitation derived linearly from the velocity boundary condition.

$$\{f\} = [T_1]\{v_n\} = [T_1][T_2]\{y\} = [T_1][T_2]\{y\} \tag{5}$$

where $[T_1]$ is the transformation matrix to convert element normal velocities $\{v_n\}$ into the exciting vector $\{f\}$, and $[T_2]$ in which a factor $(i\omega)$ has been included is the transformation matrix to convert nodal displacements $\{y\}$ into element normal velocities $\{v_n\}$.

Once Eq.(4) has been solved, the pressures at several field points (e.g., m) within the acoustic domain can be written as

$$\{p_f\} = [A_f]\{q\} \tag{6}$$

where $[A_f]$ is the matrix with m row vectors depending on the frequency, the structural surface and the locations of m field points.

Substituting Eqs.(4) and (5) into Eq.(6) results in

$$\{p_f\} = [A_f][A]^{-1}\{f\} = [A_f][A]^{-1}[T_1][T_2]\{y\} = [T]\{y\} \tag{7}$$

where $[T] = [A_f][A]^{-1}[T_1][T_2]$ is for simplicity.

After the PEM is applied to the structural random response analysis, the PSD matrix $[S_{yy}(\omega)]$ of $\{y\}$ is already decomposed in Eq.(3). By using Eq.(5), the pseudo response of $\{f\}$ can be easily obtained, $\tilde{f}(t) = \{F(\omega)\}e^{i\omega t} = [T_1][T_2]\{Y(\omega)\}e^{i\omega t}$, and that constitutes the pseudo excitation on the right-hand side of Eq.(4). Then, the pseudo response of $\{q\}$ can be computed when the PEM is used in the IBEM, and the responding acoustic PSD matrix $[S_{qq}(\omega)]$ can be computed as follows:

$$[S_{qq}(\omega)] = \{Q(\omega)\}^*\{Q(\omega)\}^{\mathrm{T}} \tag{8}$$

Similarly, the responding acoustic PSD matrix of the acoustic pressures at m field points can be computed as follows:

$$[S_{p_f p_f}(\omega)] = \{P_f(\omega)\}^*\{P_f(\omega)\}^{\mathrm{T}} \tag{9}$$

The output auto-PSD of the acoustic pressure response at field point n can be represented as a sound pressure level (SPL) in decibel via

$$SPL_n = 10\log\left(\frac{|S_{p_n p_n}|}{p_{ref}^2}\right) \tag{10}$$

where $p_{ref} = 2\times10^{-5}$ Pa is the reference acoustic pressure.

4. Acoustic pressure PSD sensitivity analysis

The sensitivity of the acoustic field pressure PSD with respect to a given structural design variable can be obtained through the differentiation of the acoustic field pressure PSD, Eq.(9), with respect to a structural sizing design variable d_i

$$\frac{\partial[S_{p_f p_f}(\omega)]}{\partial d_i} = \frac{\partial\{P_f(\omega)\}^*}{\partial d_i}\{P_f(\omega)\}^{\mathrm{T}} + \{P_f(\omega)\}^*\frac{\partial\{P_f(\omega)\}^{\mathrm{T}}}{\partial d_i} = 2\{P_f(\omega)\}^*\frac{\partial\{P_f(\omega)\}^{\mathrm{T}}}{\partial d_i} \tag{11}$$

$$\frac{\partial\{P_f(\omega)\}}{\partial d_i} = \frac{\partial[T]}{\partial d_i}\{Y(\omega)\} + [T]\frac{\partial\{Y(\omega)\}}{\partial d_i} \tag{12}$$

where $\{Y(\omega)\}$ is the structural harmonic displacement response derived from the finite element analysis. The change in sizing design variable is very small compared to the wavelength in our problem, so the sensitivities of the matrix $[T]$ with respect to sizing design variable are assumed to be equal to zero, that is, they are independent of the sizing design variable. Hence Eq.(12) is simplified as follows:

$$\frac{\partial\{P_f(\omega)\}}{\partial d_i} = [T]\frac{\partial\{Y(\omega)\}}{\partial d_i} \tag{13}$$

From the above equation, it can be seen that the sensitivity of the pseudo acoustic pressure response is transformed into structural harmonic sensitivity analysis based on the PEM.

5. Numerical results and discussion

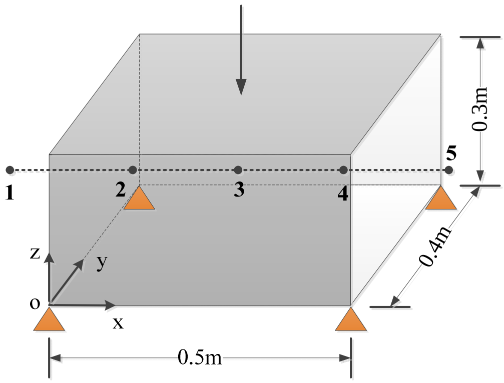

Figure 1: Structure used in the numerical simulations

In this section, as an illustrative example, the numerical results of an open box are presented to demonstrate the effectiveness of the present method in calculating the acoustic pressure PSD and its sensitivity with respect to the thickness of the open box. The open box is comprised of 4 aluminum plates: top, bottom, front and back as shown in Fig.1. The aluminum plate has a thickness of 2×10^{-3}m, Poisson's ratio 0.33, Young's modulus 6.9×10^{10}Pa, and density 2.7×10^{3}kg/m^{3}. The open box is fixed at four bottom corners. The finite element model of the box consists of 308 nodes and 506 3-node plate/shell elements. The analysis of this structural-acoustic system is conducted over the frequency range of 10-200 Hz, and the damping is not considered in the structural response analysis of this example. The length of the large element side satisfies the inquiry of the six-element-per-wavelength rule in the BEM model.

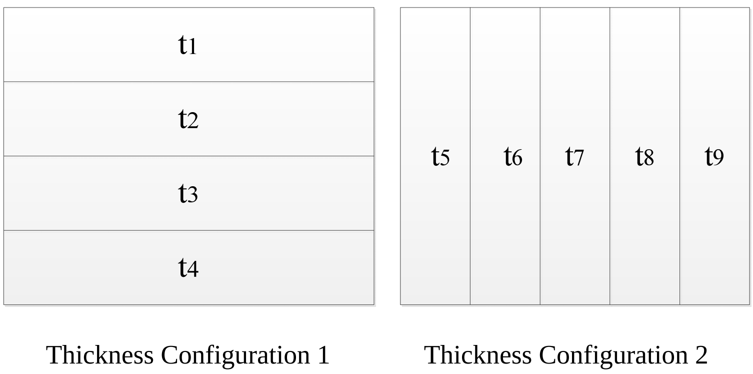

Figure 2: Thickness configurations for the top plate of the open box

A single stochastic excitations is applied on the open box's top plate. The PSD of the excitation is constant unity throughout the band frequency. The thickness of the top plate is used as the structural design variable. There are two kinds of configurations as shown in Fig.2. The first configuration consists of four strips positioned longitudinally. The thickness of the top plate at each strip is considered as an independent design variable. The second configuration consists of five strips running transversely along the top plate, and the corresponding plate thicknesses of the strip constitute the design variables. A uniform thickness 0.003m is considered as the initial thickness for all sensitivity computations. There are five field points considered, which are shown in Fig.2. All the five field points 1-5 are located at (-0.25, 0.20, 0.15) m, (0, 0.20, 0.15) m, (0.25, 0.20, 0.15) m, (0.50, 0.20, 0.15) m and (0.75, 0.20, 0.15) m respectively.

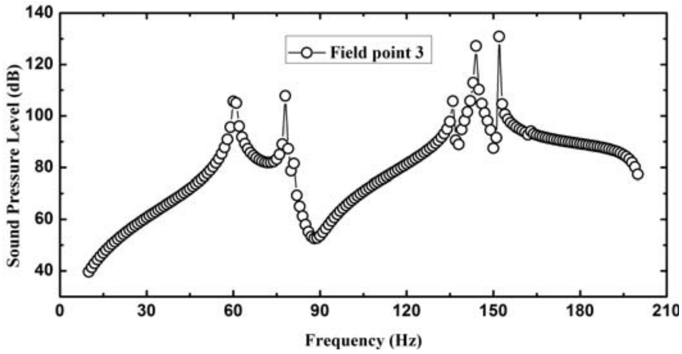

Figure 3: SPL response of the open box at field point 3 for the mid-point excitation

An acoustic pressure spectral density curve is obtained in Fig. 3 that shows the results of the analyzed frequency band. In Fig. 3, the acoustic pressure spectral density is the equivalent decibel value calculated from Eq.(10). Not all the natural frequencies of the open box are observed in the results of the analyzed frequency band from the curve. This is mainly due to the fact that the point at which the response is sought corresponds to a nodal point of the corresponding mode, Such as, the first system resonance (40Hz) is not stirred from the swinging back and forth mode, while the following two system resonances occur at 60 Hz and 78 Hz, and these frequencies coincide with the second and the third mode of the open box.

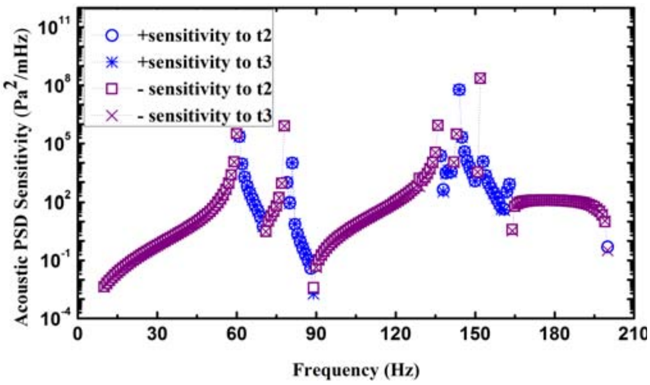

Figure 4: Acoustic PSD sensitivity values of field point 3 with respect to design variables in configuration 1

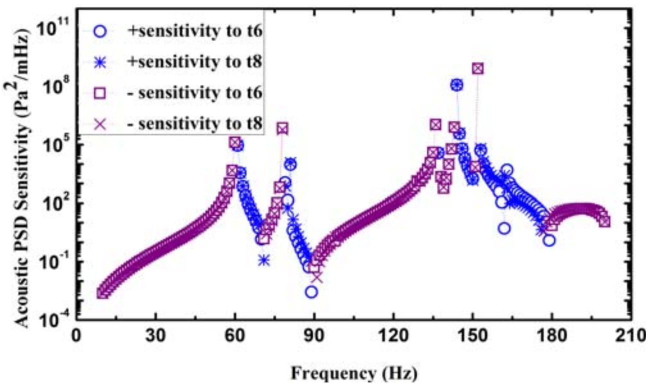

Figure 5: Acoustic PSD sensitivity values of field point 3 with respect to design variables in configuration 2

It should be pointed out firstly that the blue symbols represent positive sensitivities and the purple symbols represent negative sensitivities in all the following Figures. The acoustic PSD sensitivity values for field point 3 with respect to t2 and t3 are shown in Fig. 4, because of the symmetry of the box and that the excitation applied at the centre of the top plate, the acoustic PSD sensitivity values for field point 3 with respect to design variables t2 and t3 are expected to be the same. The acoustic PSD sensitivity values of field point 3 with respect to t6 and t8 are shown in Fig. 5, similar results can be seen and they are considered reasonable.

To further validate the sensitivity computation, the numerical results of the acoustic pressure PSD computed using the present method are compared with those calculated by using the finite difference method (FDM). The finite difference method applied here is the central difference procedure

$$p_i^{'} = \frac{p_i(\alpha + \Delta\alpha) - p_i(\alpha - \Delta\alpha)}{2\Delta\alpha} \tag{14}$$

and the step length $\Delta\alpha$ is chosen as 0.01 in Ref. [6].

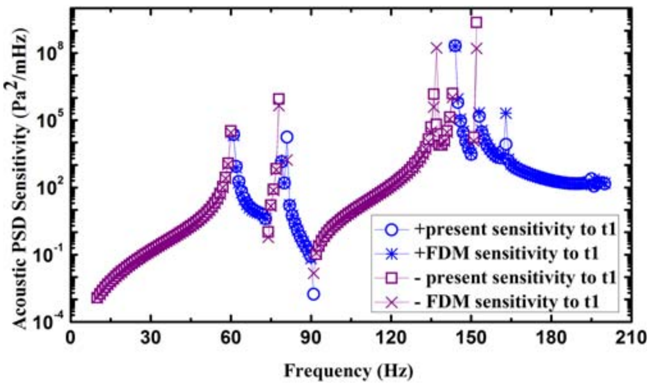

Figure 6: Acoustic PSD sensitivity values of field point 3 with respect to design variable t_1

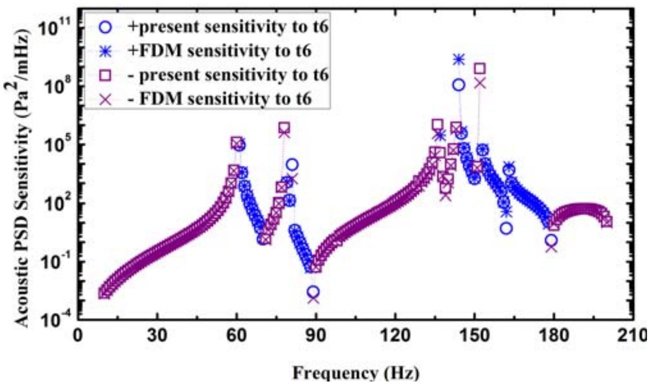

Figure 7: Acoustic PSD sensitivity values of field point 3 with respect to design variable t6

Figures 6 and 7 show the comparisons of the acoustic PSD sensitivities with respect to the design variables t1 and t6 in configurations 1 and 2, respectively, predicted using the present method and the finite difference method. As shown in both figures, overall there exists a very good correlation between the predictions by these two methods in several broad frequency bands except for some natural frequencies (This is perhaps mainly due to the neglect of the damping in the structural response analysis). As the exciting frequency increases, the modal density of the structure increases too. Hence the structure would be excited more strongly in the associated modal frequency. The differences in the computed sensitivities at some natural frequencies between these two methods represent a response shift into the very sharp acoustic resonance at these frequencies. This reflects the difficulty in capturing accurately the changes when moving in an extremely narrow resonance peak. These differences were also observed in the results for other design variables.

6. Conclusions

In this paper, a new method is developed to solve random acoustic radiation problems. The acoustic pressure PSD and its sensitivity of a randomly excited structure are investigated based on FEM and IBEM combined with PEM. When the PEM is applied to random acoustic radiation problems, the random response is transformed into the harmonic response, and the sensitivity analysis of the random response is transformed into that of the harmonic response. The formula for computing the sensitivity for a structural acoustic radiation random response is derived. The sensitivities of acoustic response with respect to structural design variables are calculated for one example and validated by comparing with the results using the finite difference method. The present integrated FEM/IBEM combined with PEM procedure provides an efficient and convenient method for engineers to solve acoustic radiation problems under stationary stochastic excitations.

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