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Structural Damage Identification by Means of Neural Network (Evaluation of Identification Capability)

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1. Abstract
Damage identification of structural system can be dealt with by means of an inverse problem approach; that is, the location as well as the magnitude of damage is determined by inversely solving the relationship between the damage and the corresponding change in structural characteristics. In this study, we adopt a definite number of natural frequencies as such structural characteristics. A multi-layered neural network approach based on an alternative error back-propagation with fixed connection weights is used to solve the inverse problem. The damage identification based on change in natural frequencies inherently has ill-posed nature. We carry out a comprehensive simulation study and discuss the capability of the proposed damage identification approach.
2. Keywords: structural health monitoring, inverse problem, neural network, natural frequencies, truss.

3. Introduction

Damage of a structural system has to be identified in order to repair or replace the damaged parts or members to maintain its original system performance. This research field is referred to as non-destructive testing, structural damage identification, or structural health monitoring. There are several approaches for the purpose. For example, the eddy current approach[1] and the ultrasonic approach[2] are typical examples of non-destructing testing. Giurgiutiu et al.[3] use piezoelectric devices to detect crack or defect. Approaches based on change in dynamic characteristics such as the natural frequencies of the target structure have also been studied [4]-[6].
We deal with the damage identification based on the change in dynamic characteristics of the entire structure caused by the damage on its constituent parts; the natural frequencies of the target structure are also adopted in this study. The relation between the local damage and the change in natural frequencies is implemented as a multi-layered neural network[7]. The error back-propagation technique with fixed connection weight [8][9] is adopted to solve the inverse problem of determining the location and magnitude of the damage corresponding to the given change in natural frequencies.
The inverse problem to be solved for the damage identification is a typical ill-posed problem in such a case, since it is fundamentally impossible to identify all of the patterns of the damaged parts based only on a limited number of natural frequencies[4]. The damage identification approach based on neural network, however, works fairly well[10]; it has been demonstrated with examples that damaged members of truss structure are successfully identified based only on its limited number of natural frequencies, in the case that the number of damaged members is small.
In the current study, we carry out comprehensive simulations in order to examine the capability of this damage identification approach based on multi-layered neural network. On the basis of the numerical experiments with truss structures, we investigate the characteristics of availability of the approach. We take into account the following: such as the size of the neural network, the constituents of the learning data set, and so on.

4. Inverse Problem Formulation of Damage Identification

There are two types of damage identification approaches. One is to identify directly the location as well as the magnitude of the damage by means of installed sensing devices or scanning equipments. The other is the indirect approach, which deduces or estimates the location and magnitude of the damage based on the observed change in some characteristics of the entire target structure. We deal with the latter approach from the inverse problem point of view.

4.1. Basic Idea

Let $\mathbf{x}$ be the state variable vector corresponding to the condition of parts or members of the target structure, such as thickness of structural elements or stiffness of truss members. Let $\mathbf{y}$ be the characteristics vector of the entire structure affected by the structural damage, which is expected to be obtained by means of some measurement devices. The relationship between the two variables can be expressed in the following general form:

$$\mathbf{y} = \mathbf{y}(\mathbf{x}). \tag{1}$$

In the case that we have a structural characteristic value $\mathbf{y}^*$ which is obtained by means of the measurement, the corresponding damaged structural condition $\mathbf{x}^*$ can be determined by solving the following problem:

$$\text{Find} \quad \mathbf{x}^* \quad \text{such that} \quad \mathbf{y}(\mathbf{x}^*) = \mathbf{y}^*, \tag{2}$$

which is a typical inverse problem based on Eq.(1). This is the basic idea of our damage identification approach.

4.2 Adopting Natural Frequencies as Key Structural Characteristics
This damage identification approach significantly depends on the type of the structural characteristics $\mathbf{y}$ in Eq.(1). We adopt the change in natural frequencies of the entire structure due to damage as such structural characteristics, in this study. There are following reasons for this selection:

- the values can be numerically calculated for various damage patterns taken into consideration
- damage of any structural elements can affect these characteristics of the entire structure, to some extent
- the values are obtained by means of a non-invasive measurement in general; application to existing structures can be possible

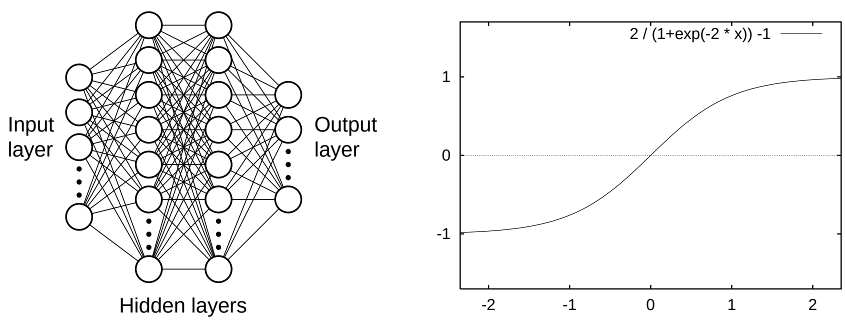
One of the issues in the case of adopting the natural frequencies for this purpose is that the inverse problem (2) becomes inherently ill-posed since the order of natural frequencies to be measured is less than the number of structural elements for damage identification in general. The aim of the current study is to investigate the feasibility of the following neural network approach based on the natural frequencies.

5. A Multi-Layered Neural Network Approach
A multi-layered neural network has the ability to acquire its input-output relationship by the error back-propagation learning[7]. The alternative error back-propagation with fixed connection-weights[8] is adopted to solve the inverse problem based on the acquired relationship. In order to make the article self-contained, we summarize this approach to solve an inverse problem by means of multi-layered neural network.

5.1 Multi-Layered Neural Network and Error Back-Propagation Learning
Figure 1(a) shows a conceptual illustration of multi-layered neural network. Neurons are depicted as circles and connected in a layered manner. The input and output layers placed at leftmost and rightmost sides are the 0th and Nth layers; the so-called hidden neurons are placed between them and make up the 1st$\cdots(N-1)$th layers. The behavior of the ith neuron of the kth layer is modeled as follows:

$$v_i^k = f(u_i^k), \quad u_i^k = \sum_j w_{ij}^k v_j^{k-1}, \quad f(u_i^k) = \frac{2}{1+\exp(-2u_i^k)} - 1, \tag{3}$$

where u_i^k and v_i^k are the internal state and the output value of the neuron and w_{ij}^k is the connection weight between the ith neuron of the kth layer and the jth neuron of the $(k-1)$th layer. The characteristic function of a neuron



(a) (b)

Figure 1: Multi-layered neural network

is expressed as f, which is shown in Fig.1(b). The input-output relation as the entire neural network can be represented in the following form:

$$\mathbf{v}^N = \mathbf{v}^N(\mathbf{v}^0, \mathbf{w}) \tag{4}$$

where $\mathbf{v}^0 = [v_i^0]$ and $\mathbf{v}^N = [v_i^N]$ are the input and output vectors. The connection weights $\mathbf{w} = [w_{ij}^k]$ determines the input-output relation.
On the basis of the error function to the desired output value $\underline{\mathbf{v}}^N = [\underline{v}_i^N]$ expressed as

$$E = \frac{1}{2}(\mathbf{v}^N - \underline{\mathbf{v}}^N)^T(\mathbf{v}^N - \underline{\mathbf{v}}^N) = \frac{1}{2}\sum_i (v_i^N - \underline{v}_i^N)^2, \tag{5}$$

the following iterative modification of the connection weights are conducted for various input values and their corresponding desired output values:

$$w_{ij}^k \leftarrow w_{ij}^k - \Delta w_{ij}^k, \quad \Delta w_{ij}^k = \varepsilon \frac{\partial E}{\partial w_{ij}^k} \tag{6}$$

where ε is an adequate small coefficient. The modification Δw_{ij}^k is obtained in the following manner:

$$\Delta w_{ij}^k = \Delta v_i^k \cdot f'(u_i^k) \cdot v_j^{k-1}, \tag{7}$$

$$\Delta v_i^k = \varepsilon \frac{\partial E}{\partial v_i^k} = \sum_j \Delta v_j^{k+1} \cdot f'(u_j^{k+1}) \cdot w_{ji}^{k+1}, \tag{8}$$

$$\Delta v_i^N = \varepsilon \frac{\partial E}{\partial v_i^N} = \varepsilon(v_i^N - \underline{v}_i^N). \tag{9}$$

Equation (8) denotes the error back-propagation process from the $(k+1)$th layer to the kth layer.

5.2 Solving Inverse Problem by Means of Alternative Error Back-Propagation
We deal with the inverse problem (2) in terms of the following error minimization problem:

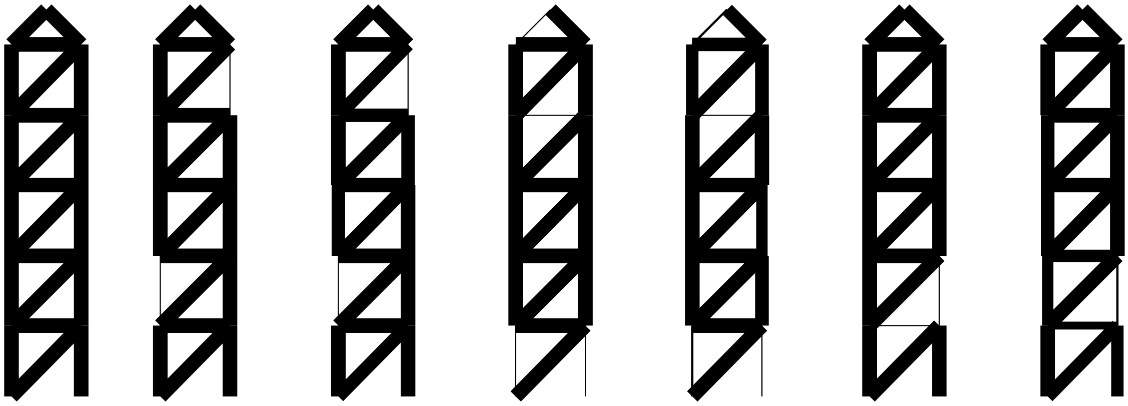
$$\text{Minimize} \quad E = \frac{1}{2}(\mathbf{y}(\mathbf{x}) - \mathbf{y}^*)^T(\mathbf{y}(\mathbf{x}) - \mathbf{y}^*) \quad \text{with respect to} \quad \mathbf{x}. \tag{10}$$

The solution can be obtained by means of an iterative gradient procedure as

$$\mathbf{x} \leftarrow \mathbf{x} - \Delta\mathbf{x}, \quad \Delta\mathbf{x} = \varepsilon \frac{\partial E}{\partial \mathbf{x}} \tag{11}$$

based on an adequate initial value of $\mathbf{x}$. After the input-output relationship (4) of the neural network has been trained to represent the intended relationship (1), the gradient $\Delta\mathbf{x} = \varepsilon(\partial E/\partial \mathbf{x})$ can be obtained as the modification to the input layer at the neural network, $\Delta\mathbf{v}^0 = \varepsilon(\partial E/\partial \mathbf{v}^0)$, in terms of Eq.(8) as the result of the error back-propagation process, since the error function (5) for the learning and the error in the minimization problem (10) to solve the inverse problem are essentially the same at this point. This enables us to solve the inverse problem (2) in terms of the minimization problem (10) by means of the neural network. Note that the connection weights of the neural network are fixed in the case of the solving process of the inverse problem, so that the implemented input-output relation should not be changed.
5.3 Damage Identification Based on Natural Frequencies by Means of Neural Network
The damage identification approach based on the change in natural frequencies by means of a multi-layered neural network is performed as follows. First, the learning data set of various damages on the target structure and the corresponding change in natural frequencies has to be prepared. This can be performed by means of numerical calculation. Second, an adequate neural network is prepared and trained to represent the intended relationship with the learning data set. After the learning process of the neural network is completed, it can be used to identify the damage of the target structure based on the measured change in natural frequencies by means of the alternative error back-propagation.

6. Comprehensive Simulation Study with Truss Structures
A number of numerical experiments are carried out in order to evaluate the capability of the proposed damage identification approach.



(a) Truss (b1) Damage assumed (b2) Identified (error 0.01%) (c1) Damage assumed (c2) Identified (error 1.00%) (d1) Damage assumed (d2) Identified (error 2.76%)
(b) Correctly identified (c) Correctly identified (d) Incorrectly identified

Figure 2: Example damage identification results based on 5-unit truss (magnitude of damage exaggerated)

6.1 Simulation Condition

Figure 2(a) shows the 2D truss structure adopted in this simulation study. The square truss units are 1m$\times$1m; all the truss members are steel rod of 10mm diameter; a mass of 1kg is attached to the top node of the truss. Damage is given in the form of degradation of stiffness of truss members; the maximum magnitude of the damage is assumed to be 10% of the stiffness. Learning data sets for the neural networks are prepared by means of numerical calculation of natural frequencies based on randomly generated various damage conditions. In the current study, the number of back-propagation learning is 500 million for all of the cases, which is considered to be enough for the convergence. The damage identification capability of the approach is examined based on all of the 5% stiffness damage combinations of 1 to 4 truss members: that is, ${}_{22}C_1 = 22$ single-member-damage patterns, ${}_{22}C_2 = 231$ two-member-damage patterns, ${}_{22}C_3 = 1540$ three-member-damage patterns, ${}_{22}C_4 = 7315$ four-member-damage patterns and 9108 patterns as the total. In these numerical experiments, an obtained damage identification result is judged adequate in the case that the maximum error in the estimated member stiffness is less than 1%, that is, the member stiffness is 94% to 96% of the original stiffness for the damaged members and 99% to 100% for the non-damaged members.

6.2 Evaluation of Damage Identification Capability

Table 1 summarizes the damage identification results based on the approach with various neural networks. On the basis of the results shown in Table 1, it is clear that the number of modes of natural frequencies has significant influence on the damage identification capability. The results based on 5 natural frequencies show that even in the case of single member damage, it is only 40% to 55% of the patterns (9 to 12 within 22 members) that are adequately identified. In the case of 8 natural frequencies, the damage identification results are significantly improved and 86% to 95% of the patterns (19 to 21 within 22 members) are adequately identified in the case of single member damage; about half are adequately identified even in the case of two member damage patterns based on the neural network having learned the data set consisting of 50,000 patterns of two member damage. In the case of 10 natural frequencies, all of the single member damage patterns are adequately identified; more than 80% of two member damage patterns and about 60% of three member damage patterns are adequately identified based on the neural networks having learned the data set consisting of two or three member damage patterns.

Composition of the learning data set should also be taken into consideration. In the case of neural networks having only learned the data set of single member damage patterns, it is shown that the identification capability is significantly limited even in the case of neural network based on 10 natural frequencies. This is considered to be because such data set does not reflect the interrelation between the damages on different truss members. The neural networks having learned two or three member damage patterns exhibit comparable identification capabilities. It is interesting that the neural networks based on three member damage patterns show marginally lower performance than those based on two member damage patterns in any case, regardless that the three member patterns are fundamentally extensions of the two member patterns.

The simulation studies are carried out based on neural networks of various sizes, that is, neural networks consisting of different numbers of neurons of hidden layers. On the basis of the results, it is demonstrated that larger neural networks have superior capabilities than smaller networks as a matter of course; however, this improvement

Table 1: Damage identification results

learning data set			neurons of	adequately identified patterns (%)				
natural frequencies	damaged members	number of patterns	hidden layers	total 1-4	case of damaged number			
					1	2	3	4
5	1	1,000	25, 25	0.4	50.0	7.8	0.5	0.0
5	1	1,000	50, 50	0.4	54.5	10.0	0.3	0.0
5	2	50,000	10, 10	0.7	40.9	9.1	1.4	0.2
5	2	50,000	25, 25	0.7	40.9	8.7	1.2	0.2
5	2	50,000	50, 50	0.7	40.9	9.1	1.4	0.2
5	2	50,000	80, 80	0.7	40.9	9.1	1.4	0.2
5	3	100,000	50, 50	0.7	40.9	7.8	1.2	0.2
5	3	100,000	100, 100	0.7	40.9	8.2	1.3	0.2
8	1	1,000	50, 50	1.5	95.5	38.1	1.6	0.0
8	2	50,000	50, 50	10.6	90.9	50.6	21.4	6.8
8	2	50,000	100, 100	11.5	90.9	51.9	22.5	7.7
8	3	100,000	50, 50	9.2	86.4	44.2	18.0	6.0
8	3	100,000	100, 100	9.6	90.9	46.8	18.7	6.2
10	1	1,000	50, 50	0.9	100.0	26.0	0.0	0.0
10	2	50,000	10, 10	26.2	100.0	81.0	45.8	20.1
10	2	50,000	25, 25	36.4	100.0	86.1	57.9	30.1
10	2	50,000	50, 50	38.7	100.0	86.1	59.3	32.7
10	2	50,000	80, 80	40.1	100.0	86.6	60.7	34.1
10	3	100,000	50, 50	36.0	100.0	83.1	55.5	30.2
10	3	100,000	80, 80	35.8	100.0	83.5	55.2	30.0
10	3	100,000	100, 100	37.0	100.0	83.5	56.3	31.3

converges at certain sizes. This indicates that the size of the neural network is not insignificant but the data set to be learned is definitely significant in this damage identification approach.
Figures 2(b)-(d) show typical examples of damage identification results based on the neural network consisting of two hidden layers of 50 neurons having learned the data set composed of patterns of 10 natural frequencies corresponding to two damaged members. The member stiffness damage assumed for these examples is 5%; the magnitude of damage in the figures is exaggerated based on this values. Figure 2(b) shows an accurate result. Figure 2(c) is a result judged marginally adequate; in the damage identification result of Fig.2(c2), slight degradation of stiffness is observed also in some of the non-damaged truss members such as the vertical members of the uppermost square unit. Figure 2(d) is a damage identification result judged inadequate; the assumed damaged members are two as shown in Fig.2(d1), but we can observe other degraded members around the two members as shown in Fig.2(d2).

7. Concluding Remarks

In this article, structural damage identification was dealt with from an inverse problem point of view. The natural frequencies of the entire structure are adopted as the key structural characteristics for the damage identification; the multi-layered neural network is used to represent the required relationship and to solve the inverse problem based on the relation, by means of the alternative error back-propagation with fixed connection weights. The approach is applied to a damage identification problem of a truss structure and simulation studies are carried out in order to evaluate its damage identification capability.
On the basis of the numerical experiment results with the 5-unit truss, the followings are concluded. It is theoretically impossible to identify *all* of the damage patterns of 22 truss members based only on 10 natural frequencies; however, it has been demonstrated that the neural network approach has the capability to identify quite significant part of the combination patterns in the case that the number of damaged members is limited. The damage identification capability significantly depends on the number of adopted natural frequencies. The learning data set for the neural network should include some patterns that represent the interrelation between the damaged parts.
It is obvious that the natural frequencies are not applicable to the case of identifying damage on symmetrical parts of a symmetrical structure. Some additional structural characteristics in order to cope with this limitation are the issue to be dealt with. From the viewpoint of practical application, sensing devices and signal processing methodologies should also be taken into consideration. These are the future works.

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