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Optimal Design of Wind Farm Layout and Control Strategy

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1. Abstract
Wind energy as one of the alternative energy sources is growing at a rapid rate for its property of renewability and abundancy in the current society. However, high power losses have been witnessed due to the intervention of air flow induced by the upstream wind turbines in the wind farm. Though researches show that the great power losses can be reduced through the optimal design of wind farm layout and control strategy, up to now only separate optimization of wind farm layout or control strategy, i.e., either the wind farm layout optimization with the constant wind turbine operations or the control optimization with the fixed wind turbine positions is reported in literatures. Meanwhile, even though it is convinced that the unrestricted coordinate method is superior to the grid based method for the wind farm layout optimization due to its flexibility to place wind turbines, the comparison between these two wind farm design methods is not made by considering the control optimization. Therefore, this paper aims to fill these research gaps. The combined wind farm layout plus control optimization is conducted in this paper for the first time, and the results of which are compared with the separate wind farm layout optimization and control optimization to demonstrate its effectiveness using both wind farm design methods. The comparative results show that the layout optimization is most inefficient in the optimal wind farm design. The control optimization has most stable performance almost without deviations for repeated calculations, and it is able to attain the best optimization results under 45 degree constant wind direction condition using the unrestricted coordinate method. Even though the combined layout plus control optimization is theoretically superior to the other optimizations which obtains the better results, it is apt to be stuck into the local optima while the global optima cannot be guaranteed with single calculation.

2. Keywords:
Layout optimization; Control optimization; Combined layout plus control optimization; Grid based method; Unrestricted coordinate method

3. Introduction
The exploitation of wind energy transformed into the electric power is accomplished by wind turbines placed in clusters to take full advantage of the local wind resources. Compared to the single-placed wind turbine, the dense placement of wind turbines in close proximity results in the problem of the wind shadowing from the upstream turbines to the nearby downstream ones, which is known as the wake effects or wake interventions [1]. With the reduced wind power output of the downstream wind turbines, the total wind farm power production is decreased affecting the cost competitiveness of the wind power. To alleviate the wake effects in the wind farm, great efforts have been made to the wind farm optimization study. And the wind farm layout optimization, i.e., changing the wind turbine positions is one approach to achieve this, while the optimization of wind farm control strategy, i.e., changing the wind turbine operations is another approach.

The study of wind farm layout optimization begins with Mosetti et al. [2], who applied the Genetic Algorithm (GA) to optimize the wind turbine positions for a square-shape wind farm in which the wind farm area is subdivided into 10 × 10 identical small square grids. The results indicate it has a great improvement for both total wind farm power production and the cost per unit power with the optimized wind farm layouts under all three tested wind conditions compare to the random wind farm layouts. Since then, large number of researches have been reported regarding the wind farm layout optimization problem through the employment of the other optimization algorithms or the improved wind farm models [3]. Nevertheless, it is found that they all share one same setting for the wind farm layout optimization studies in literatures, that is the uniform operation is applied for all wind turbines enabling every single wind turbine produces the maximum power for itself. However, researches show that the self-optimum wind farm control strategy is not the optimum choice for the total wind farm power output when taking the wake effect into account.

The improvement of the wind farm performance achieved by the wind farm control optimization has also been witnessed by researches. A wind tunnel test with 8 rows of 3 turbines was done by Corten [4] under constant wind speed condition. With the optimized control strategy by pitching the first row of wind turbines to the maximum angle, the total wind farm power output increase of 4.6% is identified. The Energy Research Centre of the Netherlands (ECN) conducted a full scale field test which consists of five variable speed, pitch controlled turbines of 2.5 MW and 80 m diameter in a row at a spacing of 3.8 RD. It was reported that the optimized WF power output can increase up to over 0.5% containing all wind directions [5]. And the big discrepancy of the power increase percentage between the wind tunnel test and the field test was claimed to be because the realistic wind conditions is quite different from the constant wind condition in the wind tunnel. Reference [6] was one of the few researches that computationally study the optimization of wind farm control strategy. In the research, four different wind farm cases were tested in the research and the performance increase was about 4% to 6% depending on the cases. However, all the above mentioned control optimization studies are proceed based on the fixed wind turbine positions while no combined optimization study of the wind farm layout plus control strategy is reported.

Therefore, this paper aims to fill the research gaps. The combined layout plus control optimization is performed for the first time and the results are compared with other two optimizations which included the layout optimization and control optimization using the two wind farm design methods. The comparative results are able to shed light on the effectiveness of the different types of optimization studies as well as the two design methods.

4. Description of the wind farm optimization problem

For the study of the optimal design of wind farm, the main objective is to reduce the wake power losses caused by the wake interventions between wind turbines. To incorporate the wind farm wake interference into optimization study, one of the critical procedures is to establish the wind turbine wake model using the explicit mathematical expressions. Among all the applied wake models, PARK model [7] is most widely used for the wind farm optimization due to its cost-effective property and accuracy compared to the real wind farm data.

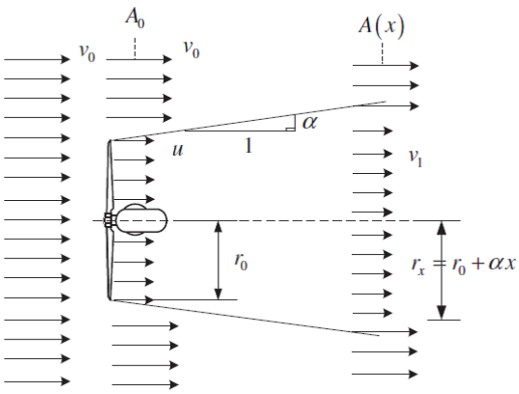

Fig. 1 Diagram of PARK wake model [8]

The PARK model assumes a linear expansion of the wake (see Fig. 1). Based on the theory of momentum conservation, the velocity in the wake of an upstream wind turbine at a distance of x towards the wind direction can be given by:

$$v_x = v_0 \left[1 - 2a \left(\frac{r_0}{r_0 + \alpha x} \right)^2 \right] \tag{1}$$

where r_0 is downstream rotor radius, α is the wake spreading coefficient, x is the proximity of the two wind turbines parallel to the wind direction, and a is axial induction denoting the percentage of wind speed decreasing from the free stream air to the air at the rotor place which is given by:

$$a = \frac{v_0 - u}{v_0} \tag{2}$$

According to the actuator disk theory [9], the single wind turbine power efficiency C_P and thrust efficiency C_T are related to the axial induction as follows:

$$\begin{aligned} C_P &= \frac{P_{\text{turbine}}}{P_{\text{wind}}} = 4a(1-a)^2 \\ C_T &= \frac{\text{Thrust Force}}{\text{Dynamic Force}} = 4a(1-a) \end{aligned} \tag{3}$$

According to the Eq (3), the theoretic maximum power efficiency can be achieved when the a is equal to 1/3, which is known as the Betz limit (C_P equals 16/27). Therefore, for only one wind turbine it should be operated at the self-optimum point to produce the maximum wind power. For multiple wind turbines in a wind farm, the traditional control strategy is to ensure every single wind turbine produces the maximum wind power of its own with a equaling 1/3. However, the self-optimum control strategy is proved to be sub-optimum when considering the wake effect, which is explained numerically in reference [6]. For the situation of more than two wind turbines in a wind farm, the total power output increase achieved by adjusting the individual control strategy for different wind turbines has been witnessed through the numerical simulation and optimization [10, 11]. However, these wind farm control optimization studies are all conducted based on the fixed wind turbine positions. And no study of the combined wind farm layout plus control optimizations has been reported in literatures, which is theoretically convinced to be able to find better results since both optimization variables are free to change.

Based on the design method that is applied for the wind farm layout optimization studies, they can be divided into two categories: the grid based method and unrestricted coordinate method. For the grid based method, fist the wind farm area is divided into a large number of identical grids and only fixed position within the cell is allowed to place the wind turbine. By employing the grid based method for wind farm layout optimization, both the placement and the number of wind turbines can be optimized during the process. For the unrestricted coordinate method, the location of each wind turbine is represented by the *X-Y* Cartesian coordinates for the two dimension region. Compared to the counterpart method, the advantage of the unrestricted coordinate method is that it helps to find better optimization results with more flexible wind turbine placements. Even though the unrestricted coordinate method is reported to be more superior to the grid based method in literatures [12, 13], from the authors' point of view the conclusion lacks the powerful evidence due to two reasons. Firstly, the coarse grid density of 10 × 10 is applied for the grid based method when making comparison, while researches indicate that the better results can be obtained with finer grids [14]. Secondly, the comparison is made with fixed self-optimal control strategies for all wind turbines, and the results maybe different when incorporating the control optimization. Therefore, it is necessary to conduct the comparative study of the two wind farm design methods in a more comprehensive manner for both wind farm layout and control optimizations.

The wind farm optimization problem studied in this paper is mathematically described in Fig. 2. The objective function of this study is the cost of per unit wind power and a traditional wind farm cost which is applied in reference [2] is employed, and the total wind farm power production is calculated as the summation of the individual wind turbine power output P_i. For the layout optimization study, the individual wind turbine power is simply related to the incoming wind speed which is calculated based on the wake deficit model described above. For the control optimization as well as the combined optimization, however, it is dependent on both the incoming wind speed and selection of axial induction value *a*. The optimization is carried out under the proximity constraint which ensures the minimum distance (chosen to be 5 wind turbine diameters in the study) between any two wind turbines to prevent from the damage. And the expressions of the proximity constraint for the two wind farm design methods are different. The wind turbine position using the grid based method is represented by the row number (*m*) and column number (*n*), while it is represented by the *x* and *y* coordinates for the unrestricted coordinate method.

Objective function:

$$\text{CoE} = N \times (\frac{2}{3} + \frac{1}{3} e^{-0.00174 N^2}) / \sum_{i=1}^{N} P_i$$

$$\text{where, } \begin{cases} P_i = 0.3 v_i^3 & \text{(for layout optimization)} \\ P_i = 2.032 a (1-a)^2 v_i^3 & \text{(for control optimization and the combined optimization)} \end{cases}$$

Constrained by:

$$\sum_{i=1}^{\text{N-1}} \sum_{j=i+1}^{\text{N}} \sqrt{\left[2.5D\left((m(j)-m(i)\right)\right]^2 + \left[2.5D\left((n(j)-n(i)\right)\right]^2} < 5D \quad \text{(for grid based method)}$$

$$\sum_{i=1}^{\text{N-1}} \sum_{j=i+1}^{\text{N}} \sqrt{\left((x(j)-x(i)\right)^2 + \left((y(j)-y(i)\right)^2} < 5D \qquad \text{(for unrestricted coordinate method)}$$

Fig. 2 Mathematical description of the wind farm optimization problem for different kinds of optimizations and different wind farm design methods

5. Results and Discussion

In order to investigate the relationship between different optimizations using the two wind farm design methods, C++ codes tailored for the different optimization studies implementing the Genetic Algorithm (GA) tool are developed for the two cases. They share the same wind farm square shape of 2 km × 2 km dimensions with flat terrain, and the wind conditions for the two cases are shown as follows:

1) The constant wind speed of 12 m/s and constant wind direction of 0 degree (from the east to the west).
2) The constant wind speed of 12 m/s and constant wind direction of 45 degree (anticlockwise rotation).

5.1 0 degree wind direction case

The three different types of wind farm optimization study are performed under the 0 degree constant wind condition in the first place. Fig. 3 (a) reports the optimization fitness values using the grid based wind farm design method while the deviations of the repeated optimization results are indicated in the bar chart. It is apparent that the wind farm layout optimization yields the worst fitness value, and the combined wind farm layout plus control optimization yields the best results. For both of the two optimizations, large deviations of the repeated optimization results are detected implying that the results are highly dependent on the repeated calculations and the best optimization results cannot be ensured with single run. For the control optimization, however, even though the results are sub-optimal, they are extremely stable for different calculations with approximately no deviations. The fitness results of the three different optimizations using the unrestricted coordinate method are reported in Fig. 3 (b). It should be noted that the optimal number of turbines obtained from the grid based method optimization is used as the midpoint value of the X axis (number of wind turbines) in the plotting of the unrestricted coordinate method optimization results. Same as the grid based method result, it can be seen that the layout optimization yields the worst results as well for the unrestricted coordinate method. And the best results are obtained for the combined layout plus control optimization. For both the two optimizations, large deviations are witnessed while the deviation of the control optimization is negligible. By comparing the fitness value results of the two design methods, it is obvious that the better results are attained for the unrestricted coordinate method (approx. 1.36×10^{-3}) in comparison to the grid based method (approx. 1.38×10^{-3}).

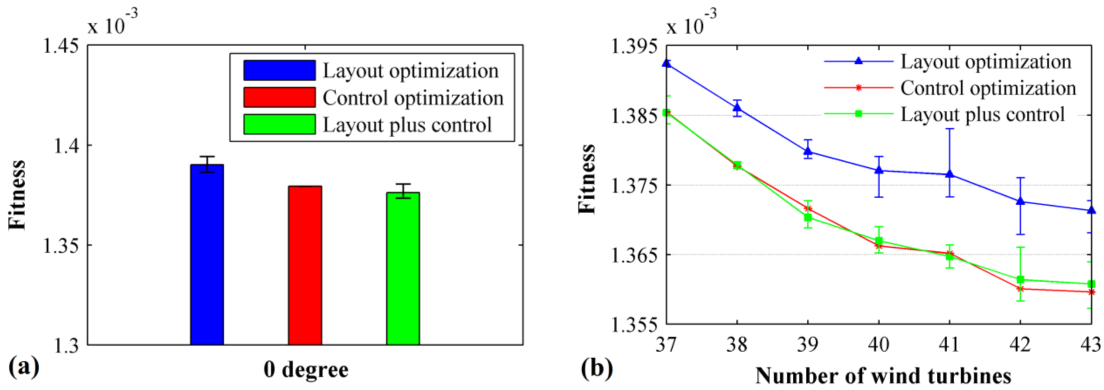

Fig. 3 Fitness value results for Layout optimization, control optimization and layout plus control optimization with (a) the grid based method and (b) the unrestricted coordinate method under 0 degree wind direction

Fig. 4 (a) reports the distribution of the wind turbine axial induction values according to the optimization results using the grid based method, and Fig. 4 (b) reports the distribution of the wind turbine axial induction values according to the optimization results using the unrestricted coordinate method. For both figures, the optimal wind farm layout is also indicated with circles denoting the wind turbines. As can be seen, most of the wind turbines are distributed along the wind farm two sides perpendicular to the wind direction. As a result, the wake interventions between wind turbines can be alleviated with enlarged distances. The leeward (downstream) wind turbines have the largest axial induction values of Betz limit 1/3, since there are no other turbines affected by them and they adopts the self-optimum control strategy to produce the maximum wind power of their own. The windward (upstream) turbines have smaller axial induction values ranging from 0.28 to 0.31 according to the optimization results using the grid based method, and ranging from 0.29 to 0.32 according to the optimization results using the unrestricted coordinate method. For the two design methods, the axial induction values of the windward turbines for the optimization using the unrestricted coordinate method are relatively bigger than that using the grid based method.

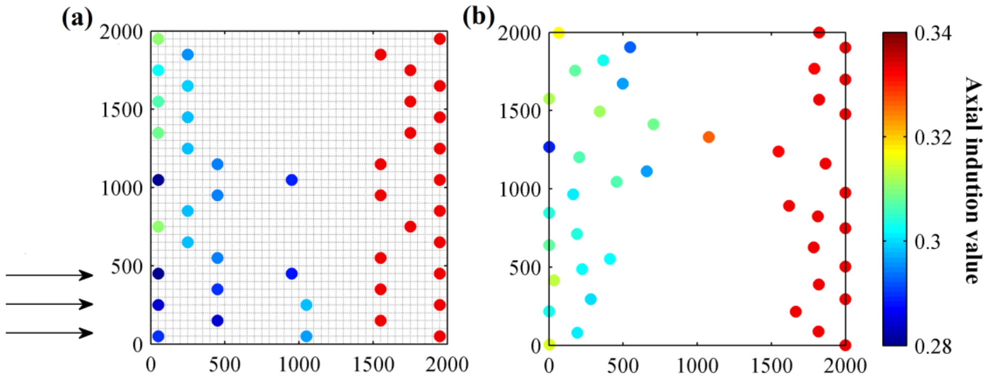

Fig. 4 Optimal wind turbine position and the optimized axial induction values with (a) the grid based method and (b) the unrestricted coordinate method under 0 degree wind direction. And the wind turbines are denoted with painted circles.

5.2 45 degree wind direction case

In this section, the different types of wind farm optimization study are performed under the constant 45 degree wind direction condition. Fig. 5 (a) reports the fitness results using the grid based wind farm design method while the deviations of the repeated optimization results are indicated. Like the results of the 0 degree wind direction case, the wind farm layout optimization yields the worst fitness value as well in this case, and the combined wind farm layout plus control optimization yields the best results. For both of the two optimizations, large deviations of the repeated optimization results are detected. For the control optimization, however, the deviations of the fitness are negligible indicating the stable optimization results for the repeated calculations. Then, the fitness results of the three different optimizations using the unrestricted coordinate method are reported in Fig. 5 (b). Like the above 0 degree wind direction case, the optimal number of turbines obtained from the grid based method optimizations is used as the midpoint value of the X axis (number of wind turbines) in the plotting of the unrestricted coordinate method optimization results. It can be seen that the layout optimization yields the worst results using the unrestricted coordinate method which has the same conclusion as the above case. Nonetheless, for all different number of wind turbines the control optimization achieves much better results than the combined layout plus control optimization in the 45 degree wind direction case, and it is because the combined optimization is stuck into the local minima.

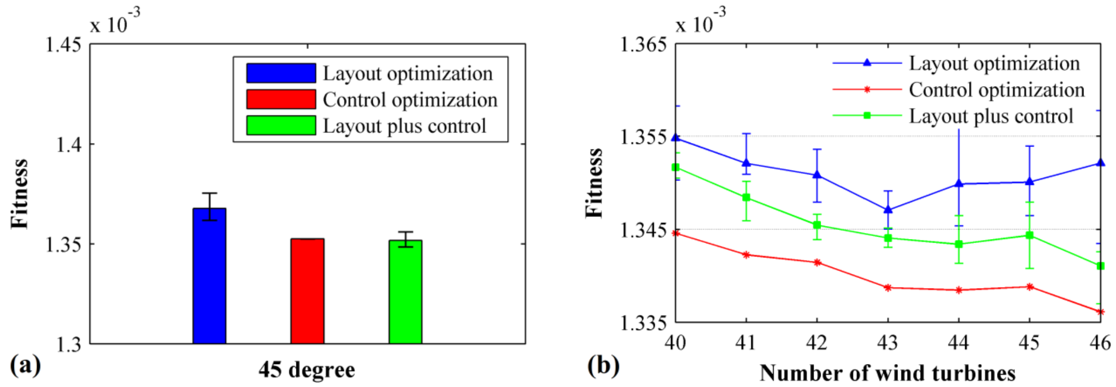

Fig. 5 Fitness value results for Layout optimization, control optimization and layout plus control optimization with (a) the grid based method and (b) the unrestricted coordinate method under45 degree wind direction

In the same manner as above case, the optimization results of the axial induction distribution under 45 degree wind direction are reported as well using both the grid based and unrestricted wind farm design methods (see Fig. 6 (a) and (b)). Unlike optimal wind farm layout under 0 degree wind direction, most of the wind turbine are scattered along all four sides of the wind farm under 45 degree wind direction. The leeward wind turbines are operated at the self-optimum point with 1/3 axial induction value while the windward turbines are operated with the axial induction value ranging from 0.29 to 0.31 for the optimization using both wind farm design methods.

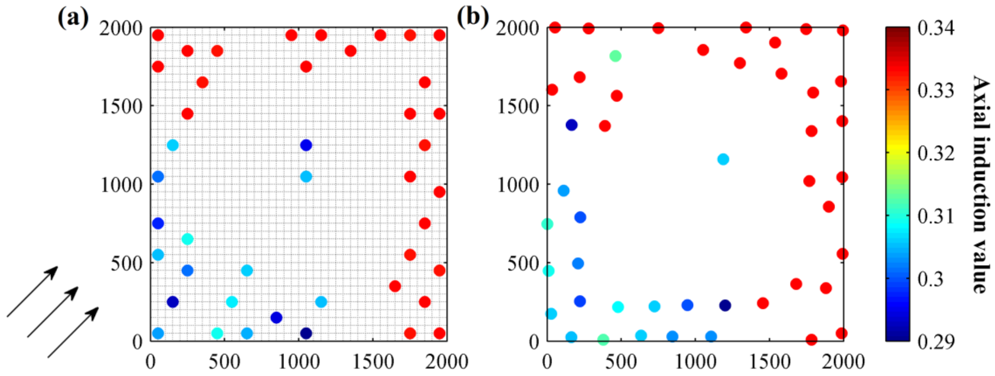

Fig. 6 Optimal wind turbine position and the optimized axial induction values with (a) the grid based method and (b) the unrestricted coordinate method under 45 degree wind direction. And the wind turbines are denoted with painted circles.

6. Conclusions

The topic of the optimal wind farm design considering both the wind turbine placements and controls is investigated in the current paper. Among which, the combined wind farm layout plus control optimization study is carried out for the first time The comparative results show that the layout optimization is most inefficient in the optimal design of the wind farm obtaining the worst optimization results with large deviations for repeated calculations. The control optimization has most stable performance and it finds the best optimization results under 45 degree wind direction condition using the unrestricted coordinate method. In comparison, even though the combined layout plus control optimization is theoretically superior to the others, it is dependent on the studied wind conditions which tends to be stuck into the local minima.

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