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# Likelihood of buckling mode interaction in shape optimisation of manufacturable cold-formed steel columns

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## 1. Abstract
This paper investigates the likelihood of buckling mode interaction in shape optimisation of manufacturable cold-formed steel columns. A literature review is carried out to examine local, distortional and global buckling mode interactions. Optimised columns available in the literature and the research outcomes previously carried out by the authors are discussed in some detail. The average elastic buckling stresses are reported herein and the need for incorporating the buckling mode interactions into shape optimisation algorithms is quantified.

**2. Keywords**: Shape optimisation; Cold-formed steel structures; Buckling mode interactions.

## 3. Introduction
Cold-formed steel (CFS) structures are widely used due to their advantages over hot-rolled steel, which include a high capacity to weight ratio, lightweight profiles, reduced installation time and manufacturing processes at room temperature [1]. The manufacturing processes, namely “roll-forming” and “brake-pressing”, allow the formation of any cross-sectional shape. Yet, CFS cross-sections used in practice are mainly limited to “Cee”, “Z” and “Σ” cross-sectional shapes, with or without intermediate stiffeners [2]. As the cross-sectional shape of the CFS profiles controls the fundamental buckling modes (local (L), distortional (D) (for open sections) and global (G)), discovering innovative and optimum cross-sectional shapes is a key element in saving material and enhancing the profitability of CFS members. The need is reinforced by the recent development of a new structural design method, the Direct Strength Method (DSM) [3], which allows designing any cross-sectional shape with the same degree of difficulty.

In the literature, shape optimisation of manufacturable CFS profiles has mainly been studied by Leng et. al. [4], Wang et. al. [5, 6] and Franco et. al. [7]. In these studies, different optimisation methods were used to achieve similar objectives. Nevertheless, the capacity of the optimised profiles was always calculated using the DSM as published in the North American [8] and Australian [9] design specifications. Therefore, only local-global (LG) buckling mode interaction was considered in the DSM equations [8, 9]. In all of the above studies, various column lengths and number of manufacturing folds per cross-section were investigated.

Results from the literature and the research performed by the authors show that, (i) the nominal member axial compressive capacity of short and intermediate columns is governed by local or distortional buckling [4], (ii) the nominal member axial compressive capacity of all columns is typically governed by global buckling [6] and (iii) the nominal distortional and global axial compressive capacities are close to each other [4, 6]. This indicates that global-distortional buckling mode interaction may occur. However, as the different buckling modes have different post-buckling reserves [10], similar level of local, distortional and global capacities does not necessarily involve interaction. The aim of this paper is to quantify if buckling mode interaction needs to be considered in the design equations in shape optimisation of CFS manufacturable columns.

In this paper, existing literature on local-distortional (LD), distortional-global (DG) and local-distortional-global (LDG) buckling mode interactions is reviewed. The corresponding DSM equations in compression are also reviewed. The average elastic buckling stresses of the optimised cross-sections available in the literature [4] and those of the authors’ previous study [6] are summarised herein and the likelihood of the buckling mode interaction occurring in shape optimisation of manufacturable CFS columns is quantified.

## 4. Existing literature on buckling mode interaction
Buckling mode interaction was shown to significantly affect the post-buckling behaviour and ultimate strength of CFS members [11-13]. Yet, only LG buckling mode interaction is currently considered in design specifications [8, 9]. Numerical and experimental investigations, such as in [11-14], are currently performed to better understand LD, DG and LDG buckling mode interactions and new DSM buckling mode interaction equations are being developed.

However, as these new equations are usually conservative when no buckling mode interaction is considered, the domain of validity of these equations is currently unclear. A summary of the DSM equations can be found in [15] and in Sections 4.1 to 4.3.

CFS columns usually experience buckling mode interaction due to close values of elastic buckling stresses [15], i.e. $f_{ol} \approx f_{od}$ (for LD buckling mode interaction), $f_{od} \approx f_{oc}$ (for DG buckling mode interaction) and $f_{ol} \approx f_{od} \approx f_{oc}$ (for LDG buckling mode interaction), where $f_{ol}$, $f_{od}$ and $f_{oc}$ are the local, distortional and global elastic buckling stresses, respectively.

Silvestre et. al. [13, 16] numerically investigated the influence of LD buckling mode interaction on CFS lipped channels. The ratio between the distortional and local elastic buckling stresses ($f_{od}$ / $f_{ol}$) was chosen between 0.9 and 1.1. The studies conclude that for columns that are not prone to distortional buckling (distortional slenderness ratio $\lambda_d \leq 1.5$) and the range of $f_{od}$ / $f_{ol}$ ratios considered, the LD interactive compressive strength is fairly accurately estimated by the DSM pure distortional nominal capacity in compression $N_{cd}$. For slender column against distortional buckling ($\lambda_d$ > 1.5), the LD interactive compressive strength can be estimated by the modified DSM equations (Eqs. (3) and (4)) presented in Section 4.1. Young et. al. [14] and Kwon et. al. [17] experimentally tested CFS lipped channels that experienced LD buckling mode interaction despite large $f_{od}$ / $f_{ol}$ ratios, ranging between 1.1 and 2.7 in [14] and 1.4 and 3.2 in [17]. In these two studies, interaction was deemed to occur due to the high yield stress of the specimens that permits the development of elastic secondary bifurcation.

Dinis and Camotim [12] studied DG buckling mode interaction and the length of the columns was so selected that $f_{od}$ = $f_{oc}$. To avoid LD buckling mode interaction, the columns were designed to have the distortional elastic buckling stress to be 20% lower than the local elastic buckling stress. This suggests that a ratio $f_{od}$ / $f_{ol}$ less than 0.8 is sufficient to prevent LD buckling mode interaction.

Dinis et. al. [11, 18] experimentally and numerically investigated LDG buckling mode interaction of CFS lipped channels and designed the profiles to ensure a strong interaction with the elastic buckling stresses, $f_{ol}$, $f_{od}$ and $f_{oc}$, of no more than 3-4% apart.

4.1 DSM equations for LD buckling mode interaction

Schafer [10] estimated the nominal capacity of CFS columns against LD buckling mode interaction by replacing the nominal yield capacity $N_y$ in the DSM equations for pure local buckling with the nominal distortional capacity $N_{cd}$. The nominal capacity in compression $N_{cld}$ for LD buckling mode interaction is then given as,

$$\begin{cases} \text{For } \lambda_{ld} > 0.776: N_{cld} = \left[1 - 0.15\left(\dfrac{N_{ol}}{N_{cd}}\right)^{0.4}\right]\left(\dfrac{N_{ol}}{N_{cd}}\right)^{0.4} N_{cd} \\ \text{For } \lambda_{ld} \leq 0.776: N_{cld} = N_{cd} \end{cases} \tag{1}$$

where $N_{ol}$ is the elastic local buckling load and $\lambda_{ld}$ is the LD non-dimensional slenderness ratio expressed as,

$$\lambda_{ld} = \sqrt{\frac{N_{cd}}{N_{ol}}} \tag{2}$$

Eq. (1) is referred to as an NLD approach. Yang and Hancock [19] adopted a similar method to Schafer [10] but replaced the nominal yield capacity $N_y$ in the DSM equation for pure distortional buckling with the nominal local capacity $N_{cl}$. The nominal capacity in compression $N_{cdl}$ for LD buckling mode interaction (referred to as the NDL approach) is then given as,

$$\begin{cases} \text{For } \lambda_{dl} > 0.561: N_{cdl} = \left[1 - 0.25\left(\dfrac{N_{od}}{N_{cl}}\right)^{0.6}\right]\left(\dfrac{N_{od}}{N_{cl}}\right)^{0.6} N_{cl} \\ \text{For } \lambda_{dl} \leq 0.561: N_{cdl} = N_{cl} \end{cases} \tag{3}$$

where $N_{od}$ is the elastic distortional buckling load and $\lambda_{dl}$ is the distortional-local (DL) non-dimensional slenderness ratio expressed as,

$$\lambda_{dl} = \sqrt{\frac{N_{cl}}{N_{od}}} \tag{4}$$

4.2 DSM equation for DG buckling mode interaction

Silvestre et. al. [20] expressed the nominal capacity in compression $N_{cde}$ for DG buckling mode interaction in a similar way to [19] and replaced the nominal local capacity $N_{cl}$ in the DSM equation for pure distortional buckling with the nominal global capacity $N_{ce}$. $N_{cde}$ is then expressed as,

$$\begin{cases} \text{For } \lambda_{de} > 0.561: N_{cde} = \left[1 - 0.25\left(\frac{N_{od}}{N_{ce}}\right)^{0.6}\right]\left(\frac{N_{od}}{N_{ce}}\right)^{0.6} N_{ce} \\ \text{For } \lambda_{de} \leq 0.561: N_{cde} = N_{ce} \end{cases} \tag{5}$$

where $\lambda_{de}$ is the DL non-dimensional slenderness ratio expressed as,

$$\lambda_{de} = \sqrt{\frac{N_{ce}}{N_{od}}} \tag{6}$$

4.3 DSM equation for LDG buckling mode interaction

Dinis et. al. [11] proposed a new DSM equation for LDG buckling mode interaction and assessed its accuracy. The nominal capacity $N_{clde}$ in compression against LDG buckling mode interaction is given as,

$$\begin{cases} \text{For } \lambda_{lde} > 0.776: N_{clde} = \left[1 - 0.15\left(\frac{N_{ol}}{N_{cde}}\right)^{0.4}\right]\left(\frac{N_{ol}}{N_{cde}}\right)^{0.4} N_{cde} \\ \text{For } \lambda_{lde} \leq 0.776: N_{clde} = N_{cde} \end{cases} \tag{7}$$

where $N_{cde}$ is the nominal capacity for DG buckling mode interaction (Eq. (5)) and $\lambda_{lde}$ is the LDG non-dimensional slenderness ratio expressed as,

$$\lambda_{lde} = \sqrt{\frac{N_{cde}}{N_{ol}}} \tag{8}$$

## 5. Results and discussion

Studies on shape optimisation of manufacturable CFS columns available in the literature [4] and that of the authors [6] are summarised in Table 1, which consists of a total of 22 studies cases. Note that the work presented in [7] is not considered herein as the elastic buckling stresses of the optimised sections are not reported. Note also that the study described in [4] includes both construction (for end-use purposes) and manufacturing constraints, and considers singly and point-symmetric cross-sections. The algorithm converges to “Cee” and “Σ” cross-sectional shapes for the 610 mm and 1,220 mm long columns, respectively, and to “Cee” and squashed “S” cross-sectional shapes for the 4,880 long columns. Although the algorithm presented in [6] considers singly-symmetric cross-sections only, it converges to closed or nearly closed “Cee” and “bean” cross-sectional shapes for all column lengths. Further the elastic buckling stresses in compression for the 4,880 mm long column in [4] are not presented and compared herein as they were not reported in [4].

Fig. 1 plots the elastic buckling stress ratios $f_{od}$ / $f_{ol}$, $f_{ol}$ / $f_{oc}$ and $f_{od}$ / $f_{oc}$ for the cases reported in Table 1. The 610 mm and 1,220 mm long columns in [4] and the 500 mm long columns in [6] usually show close local and distortional elastic buckling stresses, with $f_{ol}$ and $f_{od}$ within 20% of each other. LD buckling mode interaction is therefore likely to occur. However, for these columns, the distortional slenderness ratio is less than 1.5 and the DSM equation for pure distortional buckling is able to accurately predict the strength of the columns (see Section 4 and [9]). No DSM interaction equations therefore need to be considered for these columns. For the 1,500 mm and 3,000 mm long columns in [6], $f_{ol}$ is in general at least twice greater than $f_{od}$ and no LD buckling mode interactions is likely to occur.

In Fig. 1, the global elastic buckling stress is always greater than the distortional counterpart. For 5 out of 22 studied cases, the ratio $f_{od}$ / $f_{oc}$ is less than 0.5. Therefore DG buckling mode interaction is unlikely to occur for these cases. For the remaining cases, the $f_{od}$ / $f_{oc}$ ratio closer to unity (1.0) is 0.85 and the average $f_{od}$ / $f_{oc}$ ratio equals 0.68. Therefore, no close proximity of the global and distortional elastic buckling stresses exists for the remaining cases and the likelihood of DG buckling mode interaction is low.

Additionally, the elastic local, distortional and global buckling stresses are never close to each other, i.e. all within 15 % of each other, and LDG buckling mode interaction is therefore unlikely to be encountered.

Table 1: Summary of available studies in the literature

| Study | Yield stress $f_y$ (MPa) | Objective of algorithm | Number of folds $N_f$ | Column length (mm) | | |
|---|---|---|---|---|---|---|
| Leng et. al. [4] | 228 | Maximise the column capacity for a 279.4 mm wide and 1 mm thick steel sheet | 4, 6, 8, 10 and 12 | 610 | 1220 | 4880 |
| Wang et. al. [6] | 450 | Minimise the cross-section area of a 1.2 mm thick columns subjected to axial compressive load of 75 kN | 5, 7, 9, 11, 13 and 15 | 500 | 1500 | 3000 |

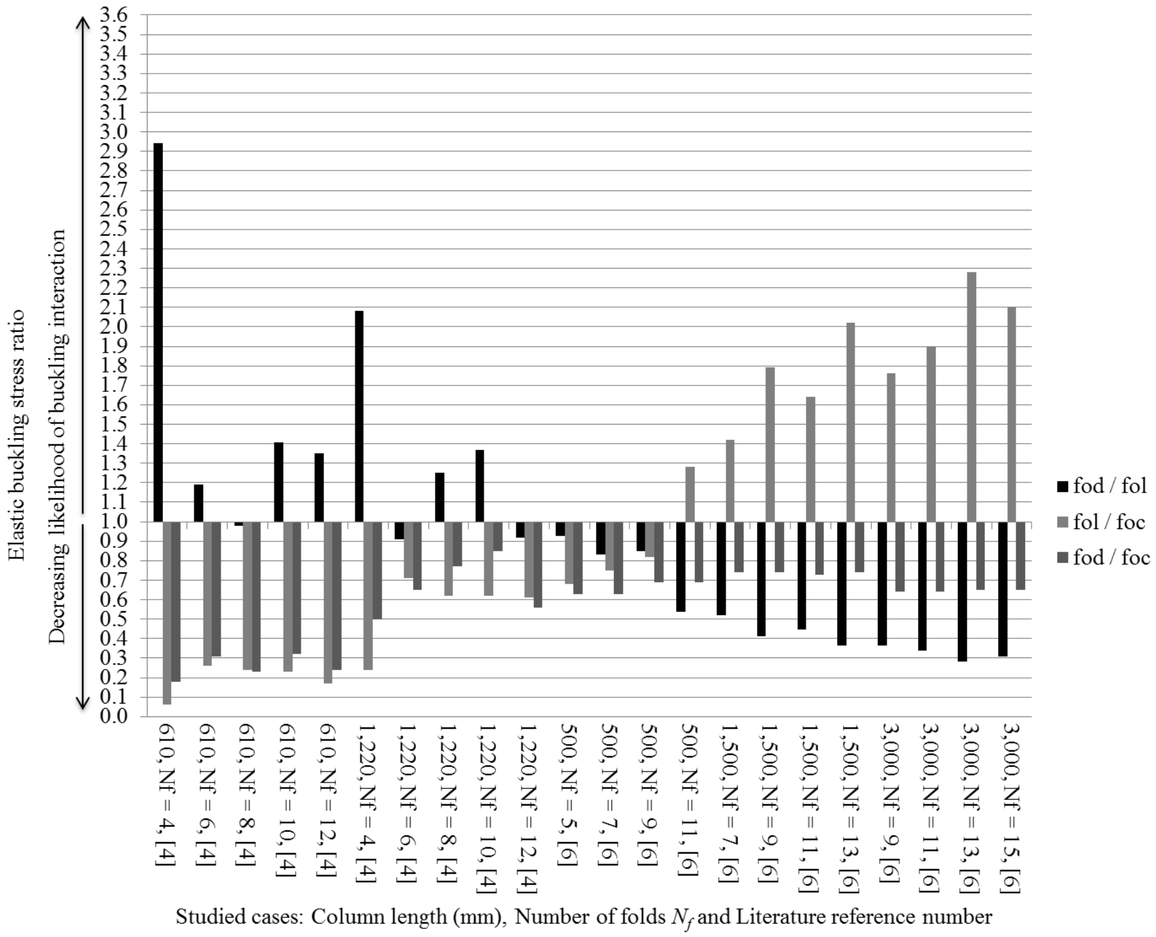

Fig. 1 Elastic buckling stress ratios for optimised columns reported in [4, 6]

**7. Conclusions**

The authors have investigated the likelihood of buckling mode interaction in shape-optimised manufacturable CFS columns. The literature on buckling mode interactions and the DSM equations proposed by various researchers to determine the nominal capacity of a column experiencing LD, DG and LDG buckling mode interaction were reviewed. Although, the nominal global and distortional capacities are close to each other, this paper shows that strong buckling mode interactions are unlikely to be encountered in the studied cases. As such, buckling mode interactions still need to be further assessed in future studies on shape-optimisation of CFS members.

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