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Tradeoff exploration in decomposition-based optimization

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1. Abstract

In multidisciplinary design, the approaches based on decomposition formally describe the design tasks by means of interconnected optimization (sub)problems, each presenting objective and constraint functions specific to different disciplines or subsystems. The presence of multiple objectives implies that tradeoffs must be taken into account during the design process, which, in engineering design, has been addressed through the concept of Pareto optimality. Although a number of numerical methods have been developed for computing Pareto solutions in applied fields, such a concept has not been extensively investigated when applied to decomposition schemes for large-scale problems.

This paper responds to the need of understanding the impact of multicriteria decision making techniques on distributed multidisciplinary optimization. To rely on Lagrangian duality, that proved to be effective for subproblem coordination in the single objective case, scalarization techniques such as the weighted-sum method, ε-constraint method, and Chebyshev-norm method, are considered in this paper. Scalarized optimization subproblems are properly formulated and a solution algorithm for computing Pareto designs, whose convergence is proven based on pre-existing results, is demonstrated on a numerical example, showing that subproblem negotiation is the mechanism that allows tradeoff exploration and the computation of Pareto designs for the overall system.

2. Keywords: Multidisciplinary optimization, multiobjective optimization, Lagrangian relaxation, Pareto design.

3. Introduction

Decomposition of optimization problems into subproblems, that has been widely used to reduce the size of the problem and exploit parallel computing, reflects from the design perspective the distribution of the tasks over different design groups according to their expertise in specific fields, that nowadays are more and more integrated in engineered systems. The integration of different disciplines underlies the presence of multiple criteria asking for negotiation mechanisms that drive the decision making process. Through negotiation it is in fact possible to reach a compromise solution that in the optimal design context is related to Pareto optimality.

Multiobjective extensions of algorithms developed for the single objective case include the Multiobjective Collaborative Optimization [1] and the Multiobjective Concurrent SubSpace Optimization [2]. The capability of genetic algorithms to converge towards Pareto (or, more appropriately, nondominated) solutions is exploited in [3, 4, 5].

A different school of thought, to which this paper belongs, relies on the previously-established results originated in the single objective scenario, in which subproblem coordination is achieved through Lagrangian relaxation. The subproblems that are identified in the MDO problem share engineering quantities that prevent separability. In order to decompose the problem, copies of the shared variables are introduced along with equality constraints between the original variables and their copies to guarantee the consistency with the original problem formulation. Decomposition becomes possible after relaxing such equality constraints into the objective function, which is unique in the single objective case. Examples of this type of approach in engineering optimization can be found in [6, 7, 8, 9]. To extend this decomposition and solving scheme based on Lagrangian relaxation to the multiobjective case, the scalarization of the multiobjective problem is necessary. While weighted-sum aggregations are proposed in [10, 11], this paper complements the previous works by discussing popular scalarization techniques other than the weighted-sum, namely the ε-constraint method and the Chebyshev-norm method.

The objective is to implement a coordination algorithm that is able to compute Pareto design making use of the negotiation mechanism that originates in the scalarization and in the subproblem communication enabled by Lagrangian relaxation. After introducing the multidisciplinary, multiobjective problem in Section 4, three scalarized, relaxed formulations are presented in Section 5. Some details about the coordination algorithm and the numerical results are reported in Section 6. A short discussion concludes the paper in Section 7.

4. Formulation of the design problem

A multidisciplinary multiobjective optimization problem is formulated as

$$\begin{aligned}
\min_{\substack{\mathbf{x}_i,\mathbf{x}^i \\ i=1,\dots,N}} \quad & \begin{bmatrix} \vdots \\ \mathbf{f}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{q}_i(\mathbf{x}^i)) \\ \vdots \end{bmatrix} \\
\text{s.t.} \quad & \mathbf{x}_i \in X_i, \mathbf{x}^i \in X^i \\
& \mathbf{g}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{q}_i(\mathbf{x}^i)) \leq \mathbf{0} \\
& i = 1,\dots,N
\end{aligned} \tag{1}$$

where $\mathbf{f}_i$ and $\mathbf{g}_i$ are the design criteria and constraints assigned to a specific team i that has control over the decision variable $\mathbf{x}_i$, and $\mathbf{p}_i(\mathbf{x}_i)$ and $\mathbf{q}_i(\mathbf{x}^i)$ are the linking variables that model the relationships between team i and the remaining teams that have control on variables $\mathbf{x}^i = [\mathbf{x}_j]_{\forall j \neq i}$ (i.e., all design variables except for those of team i), with $X^i = \prod_{j=1,j\neq i}^{N} X_j$. The linking variables collect all of the possible interactions between the teams, $\mathbf{p}_i(\mathbf{x}_i) = [\mathbf{p}_{ij}(\mathbf{x}_i)]_{\forall j\neq i}$ and $\mathbf{q}_i(\mathbf{x}^i) = [\mathbf{q}_{ij}(\mathbf{x}_j)]_{\forall j\neq i}$.
Problem (1) requires dedicated, distributed algorithms because of the dynamics involved among the design teams, which makes the problem unsuitable for centralized algorithms. The introduction of copies $\mathbf{Q}_{ij}$ and $\mathbf{P}_{ij}$ gives the teams the freedom to improve their respective objectives whereas the equality constraints, defined through the aggregations $\mathbf{P}_i = [\mathbf{P}_{ij}]_{\forall j\neq i}$ and $\mathbf{Q}_i = [\mathbf{Q}_{ij}]_{\forall j\neq i}$,

$$\begin{aligned}
\mathbf{P}_i - \mathbf{p}_i(\mathbf{x}_i) = \mathbf{0} \\
\mathbf{Q}_i - \mathbf{q}_i(\mathbf{x}^i) = \mathbf{0}
\end{aligned} \tag{2}$$

limit their freedom in the light of the decisions made by the others and, in the mathematical sense, guarantee the consistency with the original problem (1). The integration of the new variables $\mathbf{P}_i$ and $\mathbf{Q}_i$ and constraints (2) into (1) yields the following formulation

$$\begin{aligned}
\min_{\substack{\mathbf{x}_i,\mathbf{x}^i,\mathbf{P}_i,\mathbf{Q}_i \\ i=1,\dots,N}} \quad & \begin{bmatrix} \vdots \\ \mathbf{f}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i) \\ \vdots \end{bmatrix} \\
\text{s.t.} \quad & \mathbf{x}_i \in X_i, \mathbf{x}^i \in X^i, \mathbf{P}_i \in Y_i, \mathbf{Q}_i \in Z_i \\
& \mathbf{g}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i) \leq \mathbf{0} \\
& \mathbf{P}_i - \mathbf{p}_i(\mathbf{x}_i) = \mathbf{0} \\
& \mathbf{Q}_i - \mathbf{q}_i(\mathbf{x}^i) = \mathbf{0} \\
& i = 1,\dots,N
\end{aligned} \tag{3}$$

where $Y_i = \{\mathbf{p}_i(\mathbf{x}_i) : \mathbf{x}_i \in X_i\}$ and $Z_i = \{\mathbf{q}_i(\mathbf{x}^i) : \mathbf{x}^i \in X_i\}, i = 1,\dots,N$. Problem (3) is the starting point of the distribution procedure which includes three steps, scalarization, relaxation, and the actual decomposition into subproblems. Note that this order cannot be changed since scalarization yields a single objective problem suitable for Lagrangian relaxation that makes the problem additively separable.

5. Decomposition of scalarized problems

5.1. Weighted-sum scalarization

The application of weighted-sum transforms problem (3) into a single-objective problem due to the introduction of positive weights, which in the spirit of MDO can be conveniently categorized into intra-subproblem weights $\mathbf{w}_i$ that quantify the tradeoffs within each subproblem and the inter-subproblem weights b_i that balance the importance among the subproblems. The relaxation of the consistency constraints (2) in the single-objective problem yields

$$\begin{aligned}
\min_{\substack{\mathbf{x}_i,\mathbf{x}^i,\mathbf{P}_i,\mathbf{Q}_i \\ i=1,\dots,N}} \quad & \sum_{i=1}^{N} b_i \mathbf{w}_i^T \mathbf{f}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i) + \sum_{i=1}^{N} \mathbf{u}_i^T (\mathbf{P}_i - \mathbf{p}_i(\mathbf{x}_i)) + \sum_{i=1}^{N} \mathbf{v}_i^T (\mathbf{Q}_i - \mathbf{q}_i(\mathbf{x}^i)) \\
\text{s.t.} \quad & \mathbf{x}_i \in X_i, \mathbf{x}^i \in X^i, \mathbf{P}_i \in Y_i, \mathbf{Q}_i \in Z_i \\
& \mathbf{g}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i) \leq \mathbf{0} \\
& i = 1,\dots,N
\end{aligned} \tag{4}$$

which can be decomposed into N subproblems of the type

$$
\begin{aligned}
\min_{\mathbf{x}_i,\mathbf{Q}_i}\ & b_i\mathbf{w}_i^T\mathbf{f}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i)-\mathbf{u}_i^T\mathbf{p}_i(\mathbf{x}_i)+\mathbf{v}_i^T\mathbf{Q}_i \\
\text{s.t.}\quad & \mathbf{x}_i\in X_i,\mathbf{Q}_i\in Z_i \\
& \mathbf{g}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i)\le\mathbf{0}
\end{aligned}
\tag{5}
$$

where $\mathbf{u}_i,\mathbf{v}_i,i=1,\dots,N$, are the Lagrange multipilers associated with the relaxed constraints.

5.2. ε-constraint scalarization

When the ε-constraint method is applied, only one objective is optimized, arbitrarily, the s-th objective of the l-th subproblem. Tradeoffs are taken care of by bounding the remaining objectives that are placed in inequality constraints. The consistency constraints (2) are then relaxed to the optimized objective function giving the problem

$$
\begin{aligned}
\min_{\substack{\mathbf{x}_l,\mathbf{x}^l,\mathbf{P}_l,\mathbf{Q}_l\\ \mathbf{x}_i,\mathbf{x}^i,\mathbf{P}_i,\mathbf{Q}_i\\ i=1,\dots,N,i\neq l}}\ & f_l^s(\mathbf{x}_l,\mathbf{p}_l(\mathbf{x}_l),\mathbf{Q}_l)+\mathbf{u}_l^T(\mathbf{P}_l-\mathbf{p}_l(\mathbf{x}_l))+\mathbf{v}_l^T(\mathbf{Q}_l-\mathbf{q}_l(\mathbf{x}^l))+ \\
& +\sum_{i=1,i\neq l}^{N}\mathbf{u}_i^T(\mathbf{P}_i-\mathbf{p}_i(\mathbf{x}_i))+\sum_{i=1,i\neq l}^{N}\mathbf{v}_i^T(\mathbf{Q}_i-\mathbf{q}_i(\mathbf{x}^i)) \\
\text{s.t.}\quad & \mathbf{x}_l\in X_l,\mathbf{x}^l\in X^l,\mathbf{P}_l\in Y_l,\mathbf{Q}_l\in Z_l \\
& \mathbf{x}_i\in X_i,\mathbf{x}^i\in X^i,\mathbf{P}_i\in Y_i,\mathbf{Q}_i\in Z_i \\
& f_l^r(\mathbf{x}_l,\mathbf{p}_l(\mathbf{x}_l),\mathbf{Q}_l)\le\varepsilon_l^r,\ r\neq s \\
& \mathbf{f}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i)\le\varepsilon_i \\
& \mathbf{g}_l(\mathbf{x}_l,\mathbf{p}_l(\mathbf{x}_l),\mathbf{Q}_l)\le\mathbf{0} \\
& \mathbf{g}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i)\le\mathbf{0} \\
& i=1,\dots,N,\ i\neq l
\end{aligned}
\tag{6}
$$

which is decomposable into a subproblem related to team l

$$
\begin{aligned}
\min_{\mathbf{x}_l,\mathbf{Q}_l}\ & f_l^s(\mathbf{x}_l,\mathbf{p}_l(\mathbf{x}_l),\mathbf{Q}_l)-\mathbf{u}_l^T\mathbf{p}_l(\mathbf{x}_l)+\mathbf{v}_l^T\mathbf{Q}_l \\
\text{s.t.}\quad & \mathbf{x}_l\in X_l,\mathbf{Q}_l\in Z_l \\
& f_l^r(\mathbf{x}_l,\mathbf{p}_l(\mathbf{x}_l),\mathbf{Q}_l)\le\varepsilon_l^r,\ r\neq s \\
& \mathbf{g}_l(\mathbf{x}_l,\mathbf{p}_l(\mathbf{x}_l),\mathbf{Q}_l)\le\mathbf{0}
\end{aligned}
\tag{7a}
$$

and a collection of $N-1$ subproblems associated with the other teams $i=1,\dots,N,i\neq l$

$$
\begin{aligned}
\min_{\mathbf{x}_i,\mathbf{Q}_i}\ & -\mathbf{u}_i^T\mathbf{p}_i(\mathbf{x}_i)+\mathbf{v}_i^T\mathbf{Q}_i \\
\text{s.t.}\quad & \mathbf{x}_i\in X_i,\mathbf{Q}_i\in Z_i \\
& \mathbf{f}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i)\le\varepsilon_i \\
& \mathbf{g}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i)\le\mathbf{0}
\end{aligned}
\tag{7b}
$$

5.3. Chebyshev-norm scalarization

To avoid nondifferentiability issues, the min-max formulation of the Chebyshev-norm method can be re-written by minimizing the upper bound α on the objective functions. To preserve the team independence, the auxiliary variable α must be copied and the consistency constraints of the type $\alpha-\alpha_i=0,i=1,\dots,N$, are introduced and relaxed along with consistency constraints (2), producing the following formulation

$$
\begin{aligned}
\min_{\substack{\alpha,\alpha_i\\ \mathbf{x}_i,\mathbf{x}^i,\mathbf{P}_i,\mathbf{Q}_i\\ i=1,\dots,N}}\ & \alpha+\sum_{i=1}^{N}m_i(\alpha-\alpha_i)+\sum_{i=1}^{N}\mathbf{u}_i^T(\mathbf{P}_i-\mathbf{p}_i(\mathbf{x}_i))+\sum_{i=1}^{N}\mathbf{v}_i^T(\mathbf{Q}_i-\mathbf{q}_i(\mathbf{x}^i)) \\
\text{s.t.}\quad & \mathbf{x}_i\in X_i,\mathbf{x}^i\in X^i,\mathbf{P}_i\in Y_i,\mathbf{Q}_i\in Z_i \\
& b_iw_i^rf_i^r(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i)\le\alpha_i \\
& \mathbf{g}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i)\le\mathbf{0} \\
& i=1,\dots,N
\end{aligned}
\tag{8}
$$

that decomposes into N subproblems for each design task, $i = 1, \dots, N$,

$$\begin{aligned} \min_{\alpha_i, \mathbf{x}_i, \mathbf{Q}_i} \quad & -m_i \alpha_i - \mathbf{u}_i^T \mathbf{p}_i(\mathbf{x}_i) + \mathbf{v}_i^T \mathbf{Q}_i \\ \text{s.t.} \quad & \mathbf{x}_i \in X_i, \mathbf{Q}_i \in Z_i \\ & b_i w_i^r f_i^r(\mathbf{x}_i, \mathbf{p}_i(\mathbf{x}_i), \mathbf{Q}_i) \leq \alpha_i \\ & \mathbf{g}_i(\mathbf{x}_i, \mathbf{p}_i(\mathbf{x}_i), \mathbf{Q}_i) \leq \mathbf{0} \end{aligned} \tag{9a}$$

and one additional subproblem for updating the value of the auxiliary variable α

$$\min_{\alpha} \quad \alpha + \alpha \sum_{i=1}^{N} m_i \tag{9b}$$

where $m_i, i = 1, \dots, N$ are the Lagrange multipliers associated with the relaxed consistency constraint for α.

6. Numerical implementation of the coordination algorithm

The solution of the scalarized problems is obtained by an iterative procedure that is based on the duality theory. Under some mathematical conditions that are not discussed in this paper for sake of brevity (refer to [7, 9]), relaxed problems such as the ones presented in Section 5 can be solved with the following two-step iterative procedure:

1. In iteration t, solve the subproblems with respect to the so-called primal variables $\mathbf{x}_i^t$, $\mathbf{Q}_i^t$, $\mathbf{x}^{it}$, and $\mathbf{P}_i^t$ (and α_i^t) for the fixed values of the Lagrange multipliers $\mathbf{u}_i^t$, $\mathbf{v}_i^t$ (and m_i^t). In this step decomposition is exploited because the relaxed scalarized problems can be tackled by referring to subproblems, that can be independently undertaken by different teams.

2. Using the newly-computed values $\mathbf{x}_i^t$, $\mathbf{Q}_i^t$, $\mathbf{x}^{it}$, and $\mathbf{P}_i^t$, tune the multipliers (dual variables) for the dual function maximization using a subgradient update rule of the type $\mathbf{u}_i^{t+1} = \mathbf{u}_i^t + a_t(\mathbf{P}_i^t - \mathbf{p}_i(\mathbf{x}_i^t))$ (analogously for $\mathbf{v}_i$ and m_i), where a_t is an appropriate scalar. This step represents the communication between the design teams.

These steps are iterated until convergence is reached, that is, the consistency is achieved within the desired tolerance, no objective function improvement is possible, or the maximum number of iterations is reached. Convergence issues may arise due to subproblem unboundedness specifically when subproblems do not have any objective but only constraints (only Lagrangian linear terms are then minimized). For this reason quadratic penalty terms of the type $\mu \|\mathbf{Q}_i - \overline{\mathbf{q}}_i\|_2^2$ and $\mu \left\|\overline{\mathbf{P}}_i - \mathbf{p}_i(\mathbf{x}_i)\right\|_2^2$ are added. Unbounded subproblem (9b) certainly requires the quadratic augmentation $\mu \sum_{i=1}^{N} (\alpha - \overline{\alpha}_i)^2$, which leads to the finite solution $\alpha^* = \frac{\sum_i \overline{\alpha}_i}{N} - \frac{(1+\sum_i m_i)}{2\mu N}$.

This coordination algorithm has been implemented and demonstrated on an engineering problem in which the design of a suspension system for passenger vehicles is performed to optimize the ride comfort and the road holding while the team responsible for the spring design has to minimize its mass. The design team dedicated to the damper design does not have any specific objective to optimize but only design constraints to be satisfied. A representation of the problem is depicted in Fig. 1, in which the shared variables and the links between the subproblems are identified (refer to [9] for the detailed equations).

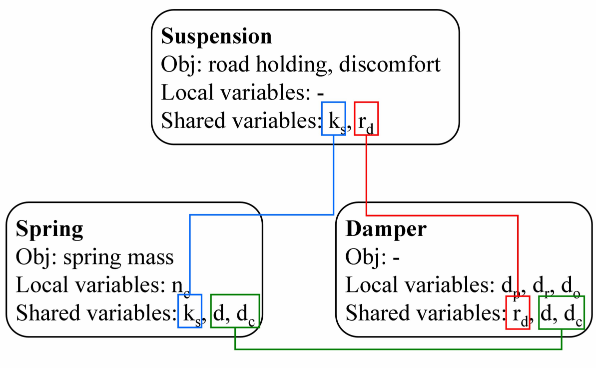

Figure 1: Model of the suspension system design problem

To verify the results of the numerical tests, the scalarized problems were solved using a centralized algorithm with different values of the scalarization parameters (weights and thresholds). The comparison showed good agreement between the results obtained with the distributed and centralized algorithms, as depicted in Fig. 2, in which the results are shown for the weighted-sum scalarization (similar plots were obtained for the other two methods and are not reported here for sake of brevity).

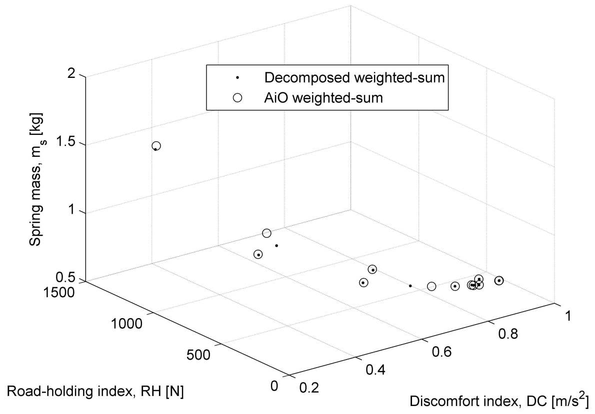

Figure 2: Pareto solutions computed with the weighted-sum method using the centralized (a.k.a., All in One, AiO) and the distributed formulations

7. Discussion

In multicriteria optimization, scalarization methods imply some sort of negotiation for setting the scalarization parameters, which allows for the computation of Pareto points as well as tradeoff exploration at those points. When these methods are applied to distributed multicriteria optimization, another layer of negotiation is added. In this paper, the distributed algorithm based on the relaxation models the communication among the design teams through the Lagrangian terms that balance the objective functions and the consistency constraints, and are intrinsically influenced by the scalarization parameters. The updating of the multipliers (and the penalty parameter in the case of quadratic augmentation), which requires the computation of new designs for each subproblem until convergence towards a (Pareto!) design that works for the whole system has been achieved, represents the inherent mechanism of negotiation in the distributed algorithm.

Relying on more advanced theory of vector-valued Lagrangian relaxation and multiobjective duality, the first layer of negotiation would be avoided, since the distributed algorithm would work on a sequence of solution sets that would be expected to converge to the Pareto set. Different formulations [12, 13] are available based on the way of relaxing the constraints over the vector of the objectives using matrices of Lagrangian multipliers U_i and V_i

$$
\begin{aligned}
\min_{\substack{\mathbf{x}_i,\mathbf{x}^i,\mathbf{P}_i,\mathbf{Q}_i\\ i=1,\ldots,N}} \quad & \begin{bmatrix} \vdots \\ \mathbf{f}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i) \\ \vdots \end{bmatrix} + \sum_{i=1}^{N} U_i(\mathbf{P}_i - \mathbf{p}_i(\mathbf{x}_i)) + \sum_{i=1}^{N} V_i(\mathbf{Q}_i - \mathbf{q}_i(\mathbf{x}^i)) \\
\text{s.t.} \quad & \mathbf{x}_i \in X_i, \mathbf{x}^i \in X^i, \mathbf{P}_i \in Y_i, \mathbf{Q}_i \in Z_i \\
& \mathbf{g}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i) \leq \mathbf{0} \\
& i = 1,\ldots,N
\end{aligned}
\tag{10}
$$

which may suggest additive separability of the type

$$
\begin{aligned}
\min_{\substack{\mathbf{x}_i,\mathbf{Q}_i\\ i=1,\ldots,N}} \quad & \sum_{i=1}^{N} \begin{bmatrix} \mathbf{0} \\ \mathbf{f}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i) \\ \mathbf{0} \end{bmatrix} - U_i\mathbf{p}_i(\mathbf{x}_i) + V_i\mathbf{Q}_i \\
\text{s.t.} \quad & \mathbf{x}_i \in X_i, \mathbf{Q}_i \in Z_i \\
& \mathbf{g}_i(\mathbf{x}_i,\mathbf{p}_i(\mathbf{x}_i),\mathbf{Q}_i) \leq \mathbf{0} \\
& i = 1,\ldots,N
\end{aligned}
\tag{11}
$$

Although multiobjective duality theory has been investigated from a theoretical standpoint, it is not ready yet for practical applications and presents two research challenges in MDO. First, since Lagrangian matrix multipliers balance the objectives and the consistency constraints, they necessarily assume different values at each Pareto point and hence result in a set of matrix multipliers U_i and V_i associated with the Pareto set; thus, a subgradient algorithm for dual vector problems is needed for tuning these matrices. Second, unlike the single-objective scenario in which $\min(\sum_i f_i) = \sum_i \min f_i$, the vector extension $\min(\sum_i \mathbf{f}_i) = \sum_i \min \mathbf{f}_i$ is not immediate because the optimal solutions are sets computed according to Pareto optimality; thus, a way to decompose (11) is required to build the Pareto set starting from the Pareto sets of subproblems (see [14] on the algebra of efficient sets) considering the associated sets of matrix multipliers.

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