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Multi-level hierarchical MDO formulation with functional coupling satisfaction under uncertainty, application to sounding rocket design.

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1. Abstract
At early design phases, taking into account uncertainty in the optimization of a multidisciplinary system is essential to assess its optimal performances. Uncertainty Multidisciplinary Design Optimization methods aim at organizing not only the different disciplinary analyses, the uncertainty propagation and the optimization, but also the handling of interdisciplinary couplings under uncertainty. A new multi-level hierarchical MDO formulation ensuring the coupling satisfaction for all the realizations of the uncertain variables is presented in this paper. Coupling satisfaction in realizations is essential to maintain the equivalence between the coupled and decoupled UMDO formulations and therefore to ensure the physical consistency of the obtained designs. The proposed approach relies on two levels of optimization and surrogate model in order to ensure, at the convergence of the optimization problem, the coupling functional relations between the disciplines. The proposed formulation is compared to a classical MDO formulation on the design of a two stage sounding rocket.

2. Keywords: MDO formulation, Stage-Wise Decomposition formulation, Uncertainty, rocket design.

3. Introduction
Multidisciplinary Design Optimization (MDO) is a set of engineering methodologies to optimize systems modeled as a set of coupled disciplinary analyses (also called subsystem analyses). For example, a launch vehicle is customarily decomposed into interacting submodels for propulsion, aerodynamics, trajectory, mass and structure. Taking into account the different disciplines requires to model and manage the interactions between them all along the design process. Using MDO in the early design phases may improve system performances and decrease design cycle cost [1]. At these steps, the determination of the optimal system architecture requires a complete design space exploration through repeated discipline simulations. To make the exploration computationally affordable, low fidelity disciplinary analyses are mostly employed, which introduce uncertainties. Handling the uncertainties at early design phases is thus essential to efficiently characterize the optimal system performances and its feasibility because it may reduce the duration and the cost of the next design phases. Uncertainty-based Multidisciplinary Design Optimization (UMDO) aims at solving MDO problems in the presence of uncertainty. This induces several new challenges compared to deterministic MDO: uncertainty modeling, uncertainty propagation, optimization under uncertainty and interdisciplinary coupling handling under uncertainty. In this paper, we focus on the interdisciplinary coupling satisfaction as it is essential to ensure the system physical consistency.
In deterministic MDO, the interactions between the disciplines are represented by coupling variables and the system multidisciplinary consistency is described as a set of interdisciplinary equations to be satisfied. Two types of coupling handling approaches may be distinguished: the **coupled** versus the **decoupled** methods depending if the couplings are found by MultiDisciplinary Analysis (MDA) or by the system optimizer at the MDO convergence. These methods may be used within single-level or multi-level UMDO formulations. Compared to single-level formulations (*e.g. Multi Discipline Feasible* (MDF) [2], *Individual Discipline Feasible* (IDF) [2]) multi-level formulations (*e.g. Collaborative Optimization* (CO), [4], *Analytical Target Cascading* (ATC) [3]) facilitate the system level optimization by introducing additional disciplinary optimizers in order to distribute the problem complexity over different dedicated disciplinary optimizations.
In the presence of uncertainty, the coupling variables are uncertain variables. Coupled single-level UMDO formulations (Fig.1) have been proposed [5,6] based on MDF combining Crude Monte Carlo (CMC) and MDA. Whereas in deterministic MDO, for a given design there is only one set of coupling variables that satisfies the interdisciplinary coupling equations, in UMDO, the uncertain coupling variables have to satisfy the system of interdisciplinary equations for each realization of the uncertain variables. The computational cost of the coupled single-level approaches becomes prohibitive due to the required number of discipline evaluations. In literature [7,8], decoupled single and multi-level UMDO formulations have therefore been investigated to overcome this computational burden. These approaches ensure the multidisciplinary system consistency for some particular realizations (*e.g.* at the Most Probable failure Point) or for the first statistical moments (*i.e.* mean, standard

deviation) of the uncertain coupling variables. Thus, it allows to limit the number of variables that have to be handled by the optimizer, however it does not ensure the multidisciplinary consistency of the system design for all the uncertain variable realizations.

The objective of this paper is to introduce a new multi-level UMDO formulation (named Multi-level Hierarchical Optimization under uncertainty - MHOU) with functional coupling satisfaction under uncertainty (*i.e.* in realizations). The rest of the paper is organized as follows. First (section 4), the existing coupled MDF formulation under uncertainty is introduced. In a second part (section 5), MHOU formulation is introduced by using subsystem optimizers in addition to the system level optimizer in order to distribute the UMDO problem complexity over different dedicated subsystem optimizations. The proposed formulation hierarchically optimizes the whole system. It relies on the iterative construction of Polynomial Chaos Expansions (PCE) in order to represent, at the convergence of the UMDO problem, the feedback couplings between the disciplines as would MDA do. PCEs allow one to remove the feedback couplings and to introduce multi-level optimization while ensuring the physical relevance of the obtained design at the convergence. Finally, in a third part (section 6), the proposed formulation is compared to MDF on the design of a two solid stage sounding rocket.

4. Coupled approach: *Multi Discipline Feasible* (MDF) under uncertainty

The most straightforward UMDO formulation is an adaptation of the single-level deterministic MDF formulation (Fig.1) which is a *coupled approach.* It consists in ensuring the coupling satisfaction by propagating uncertainty through the disciplines with CMC and to solve the system of interdisciplinary equations by MDA for each realization generated using CMC [5,6]. MDF under uncertainty can be formulated as follows:

$$\min_{\boldsymbol{z}\in\boldsymbol{\mathcal{Z}}} \Xi[f(\boldsymbol{z},\boldsymbol{Y}(\boldsymbol{z},\boldsymbol{U}),\boldsymbol{U})] \tag{1}$$

$$\text{s.t. } \mathbb{K}[\boldsymbol{g}(\boldsymbol{z},\boldsymbol{Y}(\boldsymbol{z},\boldsymbol{U}),\boldsymbol{U})] \leq 0 \tag{2}$$

with, $\boldsymbol{z}$ the design variable vector belonging to $\boldsymbol{\mathcal{Z}}$, $\boldsymbol{U}$ the uncertain variable vector defined by $\xi_{\boldsymbol{U}}(\cdot)$ the joint probability distribution function (PDF) on the sample space $\boldsymbol{\Omega}$, $f(\cdot)$ the performance function, $\Xi[\cdot]$ an uncertainty measure of the performance function (*e.g.* expected value), $\boldsymbol{g}(\cdot)$ the inequality constraint function vector and $\mathbb{K}[\cdot]$ an uncertainty measure of the inequality constraint (*e.g.* probability of failure). In the rest of the paper, an uncertain variable vector is noted $\boldsymbol{U}$ and a realization of this vector $\boldsymbol{u}$. For a given design variable vector $\boldsymbol{z}_0$, to evaluate the uncertainty measure $\Xi[f(\boldsymbol{z}_0,\boldsymbol{Y}(\boldsymbol{z},\boldsymbol{U}),\boldsymbol{U})]$, it is necessary to propagate the uncertainty in the system models. CMC is used to estimate $\Xi[\cdot]$ and $\mathbb{K}[\cdot]$. To ensure multidisciplinary system consistency, the input coupling variable vector $\boldsymbol{Y}$, which depends of $\boldsymbol{z}$ and $\boldsymbol{U}$, has to satisfy the following system of coupled equations:

$$\forall \boldsymbol{u}\in\boldsymbol{\Omega}, \forall(i,j)\in\{1,\dots,N\}^2\ i\neq j, \begin{cases}\boldsymbol{y}_{ij} = \boldsymbol{c}_{ij}(\boldsymbol{z}_i,\boldsymbol{y}_{.i},\boldsymbol{u})\\ \boldsymbol{y}_{ji} = \boldsymbol{c}_{ji}(\boldsymbol{z}_j,\boldsymbol{y}_{.j},\boldsymbol{u})\end{cases} \tag{3}$$

where N is the number of disciplines, $\boldsymbol{y}_{ij}$ is the input coupling variable vector from discipline i to discipline j, $\boldsymbol{y}_{.i}$ is the input coupling vector composed of the input coupling coming from all disciplines (represented by the dot) to the discipline i, and $\boldsymbol{c}_{ij}(\cdot)$ is the output coupling vector from discipline i to discipline j. Eqs.(3) is a system of coupled equations which has to be solved for all the realizations of the uncertain variables in order to ensure that the input coupling variables and the corresponding output coupling variables are equal. For one realization of the uncertain variable vector, the solving of Eqs.(3) is called a MultiDisciplinary Analysis (MDA) and often involves Fixed Point Iteration method. To estimate $\Xi[\cdot]$ and $\mathbb{K}[\cdot]$, repeated MDAs are performed for a set of uncertain variable realizations sampled by CMC. The computational cost of MDA under uncertainty corresponds to that of one MDA multiplied by the number of uncertain variable realizations. For each iteration in $\boldsymbol{z}$ of MDF under uncertainty, MDA is performed resulting in a prohibitive computational cost due to a large numbers of calls to the disciplines to solve Eqs.(1-2). In order to reduce the computational cost, a decoupled multi-level UMDO formulation is proposed in the following section.

5. Proposed decoupled multi-level formulation: Multi-level Hierarchical Optimization under Uncertainty

5.1. Interdisciplinary coupling satisfaction with a decoupled approach

In order to avoid the repeated MDAs, decoupled approaches aim at propagating uncertainty on decoupled disciplines allowing one to evaluate them in parallel and to ensure coupling satisfaction by introducing equality constraint in the UMDO formulation. However, two main challenges are faced to decouple the design process:

- Uncertain input coupling variable vector $\boldsymbol{Y}$ has to be handled by the system level optimizer. However, an optimizer can only handle a finite number of parameters to represent these uncertain variables. It is necessary to find a technique to represent an infinite number of realizations of the input uncertain coupling variables with a finite number of parameters.
- Equality constraints between the input coupling variables $\boldsymbol{Y}$ and the output coupling variables computed by $\boldsymbol{c}(\cdot)$, which are two uncertain variables, have to be imposed. Equality between two uncertain variables corresponds to an equality between two functions which is sometimes reduced to equality between parameters [7,8] (equality in statistical moments, in realizations, *etc.*).

In order to overcome these two issues, an approach has been proposed [9] based on a surrogate model of the coupling functional relations. In deterministic decoupled MDO approaches, considering a scalar coupling y_{ij} from the discipline i to the discipline j, only one equality constraint in the MDO formulation is added between the input coupling variable y_{ij} and the output coupling variable $y_{ij} = c_{ij}(\boldsymbol{z}_i, y_{.i})$.

However, in the presence of uncertainty, coupling satisfaction involves an equality constraint between two uncertain variables. To ensure coupling satisfaction in realizations, an infinite number of equality constraints have to be imposed, one for each realization of the uncertain variables:

$$\forall \boldsymbol{u} \in \boldsymbol{\Omega}, y_{ij} = c_{ij}(\boldsymbol{z_i}, y_{.i}, \boldsymbol{u}) \tag{4}$$

To solve this problem, an new integral form for the interdisciplinary coupling constraint is introduced:

$$J_{ij} = \int_{\boldsymbol{\Omega}} \left[c_{ij}(\boldsymbol{z}_i, y_{.i}, \boldsymbol{u}) - y_{ij}\right]^2 \xi_{\boldsymbol{U}}(\boldsymbol{u}) \mathrm{d}\boldsymbol{u} = \boldsymbol{0} \tag{5}$$

In order to have the integral in Eq.(5) equal to zero, the input coupling variables must be equal to the output coupling variables for each realization of the uncertain variables. Nevertheless, to decouple the disciplines, the uncertain input coupling variables y_{ij} have to be handled by the optimizer. In order to solve this second issue, a surrogate model of the coupling relation is introduced: $y_{ij} \to \hat{y}_{ij}\left(\boldsymbol{U}, \boldsymbol{\alpha}^{(ij)}\right)$. The surrogate model, written $\hat{y}_{ij}\left(\boldsymbol{U}, \boldsymbol{\alpha}^{(ij)}\right)$, provides a functional representation of the dependency between the uncertain variables $\boldsymbol{U}$ and the input coupling variables with $\boldsymbol{\alpha}^{(ij)}$ the vector of the metamodel parameters. To decouple the disciplines, the surrogate model parameters $\boldsymbol{\alpha}^{(ij)}$ are handled by the system level optimizer. Note that, in order to keep $\hat{y}_{ij}$ simple, the dependency between $\hat{y}_{ij}$ and $\boldsymbol{z}$ is not present here: $\hat{y}_{ij}$ is not a function of $\boldsymbol{z}$, it is learned for a specific $\boldsymbol{z}^*$ which is the unknown UMDO optimum. We propose to model the coupling functional relations with Polynomial Chaos Expansion (PCE) because this surrogate model presents advantages in terms of uncertainty analysis and propagation [10]. PCE allows one to approximate a function $h: \boldsymbol{\Omega} \to \mathbb{R}$ according to:

$$h(\boldsymbol{U}) \simeq \sum_{k=1}^{d} \alpha_k \Psi_{\mathrm{k}}(\boldsymbol{U}) = \hat{h}(\boldsymbol{U}, \boldsymbol{\alpha}) \tag{6}$$

with $\Psi_1, \cdots, \Psi_{\mathrm{d}}$ a basis of orthogonal polynomials and d the truncation degree. The choice of the polynomial basis is made consistently with the distribution $\xi_{\boldsymbol{U}}(\cdot)$ of the input random variables $\mathbf{U}$. The polynomial basis is orthogonal to the weighting function [10] of the input uncertain variable distributions. The difficulty in PCE is the estimation of the polynomial coefficients. Different techniques may be employed if black box functions are considered: the orthogonal spectral projection or the regression [10]. This approach may be easily generalized to coupling vector $\boldsymbol{y}_{ij}$. In the proposed formulation, PCE coefficients are determined with the regression method adapted to multi-level optimization. PCE is used to model the input coupling variables $\boldsymbol{Y}$ and the PCE coefficients $\boldsymbol{\alpha}$ are handled by the system level optimizer. The proposed formulation is presented in the next section.

5.2. Multi-level Hierarchical Optimization under Uncertainty (MHOU)

MHOU formulation (Fig. 2) is inspired from SWORD formulation [11] modified for uncertainty handling. MHOU is a semi-decoupled multi-level formulation ensuring interdisciplinary coupling satisfaction for all the realizations of the uncertain variables. It assumes that the system level objective $f(\cdot)$ is decomposable into a sum of the subsystem contributions. For instance, the Gross Lift-Off Weight (GLOW) of a launch vehicle is decomposable as the sum of the stage masses. In the proposed formulation, the optimization process is the following:

- At the system level:

$$\min_{\boldsymbol{z}_{sh} \in \mathcal{Z}_{sh}, \boldsymbol{\alpha}} \sum_{k=1}^{N} \Xi[f_k(\boldsymbol{z_{sh}}, \boldsymbol{z}_k^*, \widehat{\boldsymbol{y}}(\boldsymbol{U}, \boldsymbol{\alpha}), \boldsymbol{U})] \tag{7}$$

$$\text{s.t. } \mathbb{K}[\boldsymbol{g}(\boldsymbol{z_{sh}}, \boldsymbol{z}_k^*, \widehat{\boldsymbol{y}}(\boldsymbol{U}, \boldsymbol{\alpha}), \boldsymbol{U})] \le 0 \tag{8}$$

$$\forall j \in \{1, \ldots, N\}, \boldsymbol{J}_{.j}{}^* = \boldsymbol{0} \tag{9}$$

$$\forall k \in \{1, \ldots, N\}, \mathbb{K}[\boldsymbol{g}_k(\boldsymbol{z_{sh}}, \boldsymbol{z}_k^*, \widehat{\boldsymbol{y}}(\boldsymbol{U}, \boldsymbol{\alpha}), \boldsymbol{U})] \le 0 \tag{10}$$

- At the subsystem level:

$k = N$

While $k > 0$

Given $\boldsymbol{y}_{Nk}, \cdots, \boldsymbol{y}_{(k+1)k}$

For the k^{th} subsystem:

$$\mathbf{z}_{\mathrm{k}}^* = \underset{\boldsymbol{z}_k \in \mathcal{Z}_k}{\operatorname{argmin}}\, \Xi[f_k(\boldsymbol{z_{sh}}, \boldsymbol{z}_k, \widehat{\boldsymbol{y}}(\boldsymbol{U}, \boldsymbol{\alpha}), \boldsymbol{U})] \tag{11}$$

$$\text{s.t. } \mathbb{K}[\boldsymbol{g}_k(\boldsymbol{z_{sh}}, \boldsymbol{z}_k, \widehat{\boldsymbol{y}}(\boldsymbol{U}, \boldsymbol{\alpha}), \boldsymbol{U})] \le 0 \tag{12}$$

$$\forall j \in \{k+1, \ldots, N\}, \boldsymbol{J}_{kj} = \int_{\boldsymbol{\Omega}} \left[\boldsymbol{c}_{kj}\left(\boldsymbol{z_{sh}}, \boldsymbol{z}_k\, \boldsymbol{y}_{.k}\left(\boldsymbol{u}, \boldsymbol{\alpha}^{(.k)}\right), \boldsymbol{u}_k\right) - \widehat{\boldsymbol{y}}_{kj}\left(\boldsymbol{u}, \boldsymbol{\alpha}^{(kj)}\right)\right]^2 \xi_{\boldsymbol{U}}(\boldsymbol{u}) \mathrm{d}\boldsymbol{u} = \boldsymbol{0} \tag{13}$$

$k \leftarrow k - 1$

where Eq. (13) is considered for $k \neq N$. $\boldsymbol{z}_k$ is the local design variable vector of the k[th] subsystem and $\boldsymbol{z}_{sh}$ is the shared design variable vector between several subsystems. This formulation allows one to optimize each subsystem separately in a hierarchical process. The system level optimizer handles $\boldsymbol{z}_{sh}$ and the PCE coefficients $\boldsymbol{\alpha}$ of the feedback coupling variables. The handling of PCE coefficients at the system level allows one to remove the feedback couplings and to optimize the subsystems in sequence. The surrogate models of the functional feedback couplings provide the required input couplings to the different subsystems. The k[th] subsystem level optimizer handles $\boldsymbol{z}_k$ and the corresponding problem aims at minimizing the subsystem contribution to the system objective while satisfying the subsystem level constraints $\boldsymbol{g}_k(\cdot)$. The interdisciplinary coupling constraint Eq. (13) guarantees the couplings whatever the realization of the uncertain variables. This formulation is particularly suited for launch vehicle in order to decompose the design process into the different stage optimizations [11].

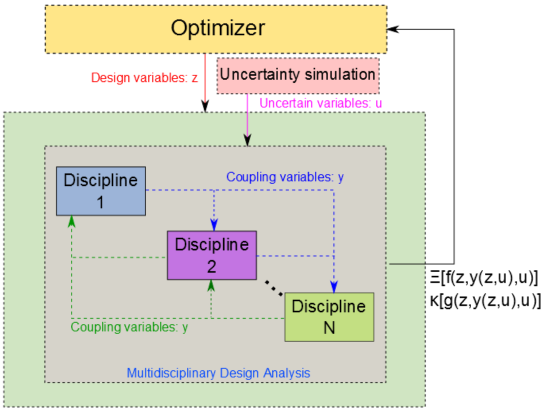

Figure 1 : MDF under uncertainty

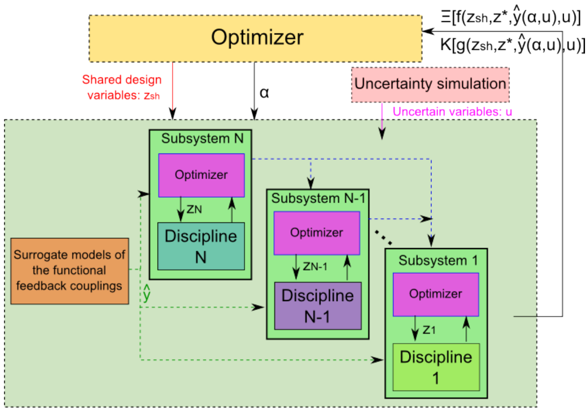

Figure 2 : MHOU formulation

6. Application: two stage sounding rocket design

The launch vehicle design test case consists of the design of a two solid stage sounding rocket for a payload of 800kg that has to reach an altitude of 300km. Four disciplines are involved: propulsion, mass budget and geometry design, aerodynamics and trajectory (Figure 3). The sounding rocket design is decomposed into two subsystems, one for each stage. MHOU formulation enables a hierarchical design process decomposed into two teams, one for each sounding rocket stage. The k[th] subsystem objective is to minimize a function of the stage mass $\mathbb{E}\left(M_k(\cdot)\right) + 2\sigma\left(M_k(\cdot)\right)$ (with σ the standard deviation). The system level objective is to minimize a function of the GLOW. The uncertainty measure for the constraints $\boldsymbol{g}_k(\cdot)$ is the probability measure $\mathbb{P}[\cdot]$. The required feedback couplings for the 2[nd] stage design are $\hat{\boldsymbol{y}}_{12} = \left[h_{f1}, v_{f1}\right]^{\mathrm{T}}$ which are the separation altitude and velocity between the 1[st] and 2[nd] stages (Fig. 3). The design constraints for the 2[nd] stage are $\boldsymbol{g}_2 = \left[Pe_2, h_{f2}, Nf_2\right]^{\mathrm{T}}$ which involve the avoidance of the breakaway of the jet in the divergent skirt (Pe_2), the apogee altitude (h_{f2}) and the maximal axial load factor (Nf_2). The same constraints are taken into account for the 1[st] stage expected for the apogee altitude. The proposed multi-level decoupled formulation and MDF under uncertainty are compared.

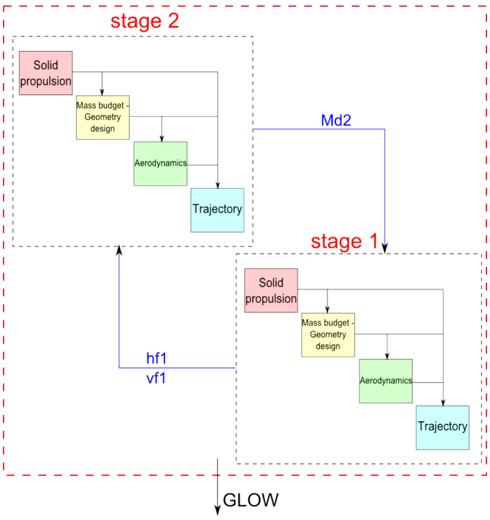

Figure 3: Design Structure Matrix of the sounding rocket

Table 1: Design variables

Design variables	Symbol	Min	Max
1[st] stage diameter (m)	D_1	0.5	1.0
1[st] stage propellant mass (kg)	Mp_1	1000	3000
1[st] stage nozzle expansion ratio	ε_1	1.0	20.0
1[st] stage grain relative length (%)	Rl_1	30	80
1[st] stage combustion depth (%)	Wp_1	30	80
2[nd] stage diameter (m)	D_2	0.5	1.0
2[nd] stage propellant mass (kg)	Mp_2	2000	3000
2[nd] stage nozzle expansion ratio	ε_2	1.0	20.0
2[nd] stage grain relative length (%)	Rl_2	30	80
2[nd] stage combustion depth (%)	Wp_2	30	80

The design variables are resumed in Table 1. The uncertain variables taken into account are the 1st stage combustion regression rate coefficient $\mathcal{N}(3.99,0.05)$ in cm/s/MPa$^{0.3}$ and the 2nd stage dry mass error $\mathcal{N}(0,50)$ in kg. The mission has to ensure that the payload reaches at least an altitude of 300km (with a probability of failure of 3×10^{-2}). MDF under uncertainty and MHOU formulations use CMC to propagate uncertainty, to estimate $\Xi[\cdot]$, $\mathbb{P}[\cdot]$, and $\boldsymbol{J}_{ij}(\cdot)$ based on a fixed set of 10^3random samples. The FPI convergence criterion is set to a relative error of 1%. Both system level optimizers are stopped when 5.6×10^6 evaluations of the disciplines is reached.

Table 2: Sounding rocket problem results

	MDF under uncertainty	MHOU
Objective function	7.07 (t)	6.68 (t)
Design variables	$[2850,0.75,4.4,69.4,66, 2395.8,0.76,9.97,69.5,65.9]^T$	$[2659.5,0.79,9.24,30.7,43.5, 2287,0.75,17.4,41.0,63.9]^T$
$\mathbb{P}[\mathrm{h_{f2}} \leq 300]$ apogee altitude	0.028	0.029

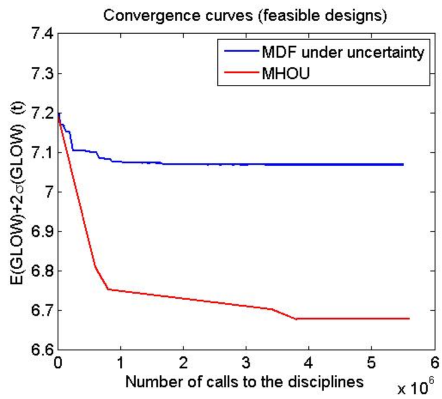

Figure 4: Convergence curves on the feasible designs w.r.t. the number of discipline evaluations

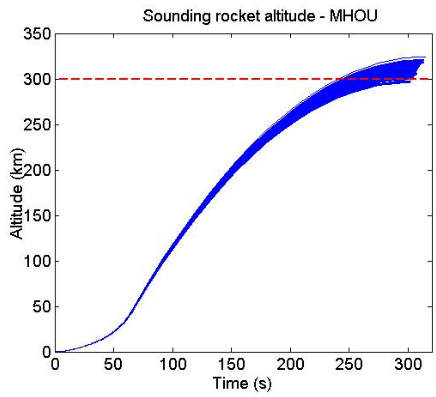

Figure 5: Sounding rocket altitude as a function of time for the optimal design - MHOU

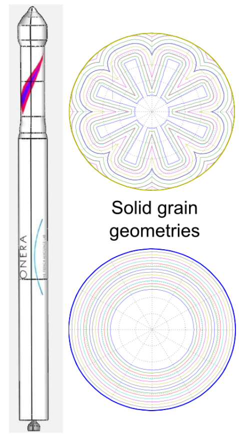

Figure 6: Optimal sounding rocket sizing and geometry

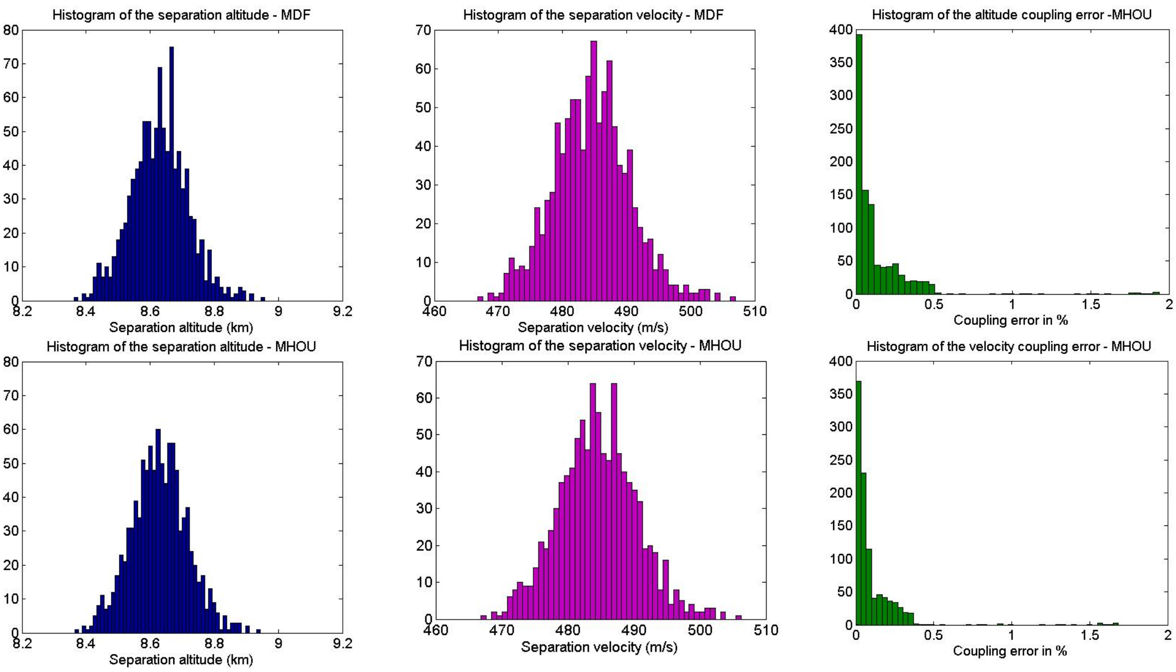

Figure 7: Altitude (blue) and velocity (pink) couplings in MDF and MHOU formulations + coupling error (green)

The discipline modeling (propulsion, sizing, aerodynamics and trajectory) is adapted from classical launch vehicle models at the early design phases [12, 13]. Gradient based optimizer (SQP) is used at the system level in both formulations. CMA-ES optimization algorithm [14] is used at the subsystem level for MHOU formulation. The two problems start from the same feasible baseline to be optimized. The results of the sounding rocket problem are resumed in Table 2. MHOU presents better characteristic in terms of quality of objective function (6.68t) than to MDF (7.07t) for a fixed discipline evaluation budget (Fig. 4). Both MDF and MHOU solutions satisfy the constraints especially the apogee altitude of 300km as illustrated in Figure 5. Only 2.9% of the trajectories do not reach the required altitude. Moreover, MHOU ensures interdisciplinary coupling satisfaction for the feedback couplings as illustrated by the comparison of the couplings for the optimal design found with MHOU with a coupled approach (MDA) and the decoupled approach (Fig. 7). The separation altitude and velocity distributions are similar with MDF (MDA on the optimal MHOU design) and MHOU (Fig. 7). Moreover, the interdisciplinary coupling error for the separation altitude and velocity are represented in Figure 7. The coupling error is always lower than 2% and concentrated between 0% and 0.5 %.

7. Conclusions: This paper provides a new multi-level hierarchical UMDO formulation that ensures the system multidisciplinary feasibility for all the realizations of the uncertain variables. The proposed formulation is based on the iterative construction of surrogate models (PCE) of the functional coupling relations. The PCE coefficients are handled by the system level optimizer and subsystem optimizers only handle local design variables. Numerical comparisons between MDF and the proposed formulation have been performed on the design of a sounding rocket highlighting the efficiency of MHOU in this test case. Additional research effort is needed to incorporate mixed aleatory and epistemic uncertainties in UMDO.

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