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Optimization of wind-induced acceleration of super tall buildings by modal shape updating

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1. Abstract

With the increase of height and flexibility of modern super tall buildings, structural wind-induced vibrations become significant under wind loads. The structural acceleration is commonly requested to be below certain limit in design process to avoid occupant discomfort under wind loads. For slightly excessive wind-induced acceleration responses scenarios, say below 20% over the code limit, the computational optimization method is usually adopted due to the fact that it's more cost effective than the supplementary damping systems method. The wind induced acceleration of certain super tall building is determined by dynamic properties under given wind loadings. As all known that the wind-induced response of super tall buildings is commonly contributed by the first vibration mode, which is actually related to the building period and modal shape of first vibration mode. A straightforward method to reduce the building acceleration is to change the natural vibration period. The natural vibration period is not quite effective due to the fact that it is global quantity and the expense of changing vibration period is very high. A modal shape updating method is proposed in this paper to reduce the building acceleration by locally calibrating the modal shape near the floor where maximum building acceleration occurs. Due to its local impact nature, the expense of changing local curve of modal shape is lower comparing with that of changing vibration period. A real super tall building project is taken as an example in the last part of this paper to show the effectiveness and applicability of the proposed modal shape updating method. The results show that the modal shape updating method provides a powerful tool for wind-induced dynamic serviceability design of super tall building structures with slightly excessive wind-induced acceleration responses.

2. Keywords: Modal shape updating, wind-induced acceleration, computational optimization, super tall buildings

3. Introduction

With the increase of height and flexibility of modern super tall buildings, structural wind-induced vibrations under wind loads have been a problem of great concern. The structural acceleration is commonly requested to be below certain limit in design process to avoid occupant discomfort under wind loads.

There are two measures to reduce the wind-induced acceleration responses when they exceed the comfort criteria. One measure is to absorb dynamic energy by installing supplementary damping systems, and the other measure is to fine-tune design variables of structural members according to structural optimization principles. When the acceleration responses exceed the comfort criteria with large margins, it is cost-effective to use dampers to mitigate peak acceleration. There are however many cases in engineering practices that the margins over the comfort criteria are small, say below 20% over the code limit, the computational optimization method is usually adopted due to the fact that it's more cost effective than the supplementary damping systems method.

For schemes of additional supplementary damping systems, lots of research have been carried out on the usage of various structural damping devices[1]. On the other hand, there are still few researches on optimization techniques for economical design of super tall buildings subject to wind-induced acceleration design requirements[2] even though wind forces and wind-induced responses can be accurately predicted by aerodynamic wind-tunnel techniques.

The wind induced acceleration of certain super tall building is determined by dynamic properties under given wind loadings. As all known that the wind-induced response of super tall buildings is commonly contributed by the first vibration mode, which is actually related to the building period and modal shape of first vibration mode. A straightforward method to reduce the building acceleration is to change the natural vibration period. The natural vibration period is not quite effective due to the fact that it is global quantity and the expense of changing vibration period is very high. A modal shape updating method is proposed in this paper to reduce the building acceleration by locally calibrating the modal shape near the floor where maximum building acceleration occurs. Due to its local impact nature, the expense of changing local curve of modal shape is lower comparing with that of changing vibration period.

4. Theoretical Basis

4.1. Wind-induced Response Analysis

It has been recognized that for many super tall buildings the across wind responses may exceed the along wind responses in terms of wind-induced dynamic serviceability design. In view of the wind-induced response of super tall buildings is commonly contributed by the first vibration mode, practical methods of acceleration response of super tall buildings are developed based on the moment gust loading factor method and the curve fitted power spectra of across wind loads[3]. By solving the equation of motion, the acceleration response equation is obtained through vibration analysis in the frequency domain as follows:

$$\sigma_{a_R(z)}^2 = \sigma_{\ddot{y}_R}^2 \approx \frac{\varphi_1^2(z)}{M_1^{*2}} \frac{\pi f_1 S_{F_1^*}(f_1)}{4(\zeta_{s1} + \zeta_{a1})} \tag{1}$$

where $\varphi_1(z)$ means the mode shape vector normalized with respect to the modal amplitude of the top floor, M_1^* means the first mode generalized mass, f_1 means the first modal frequency, ζ_{s1} means the damping ratio of the structure while ζ_{a1} for the aerodynamic damping and $S_{F_1^*}(f_1)$ means the first generalized across-wind force spectrum. The unified formula of the power spectra of across-wind loads can be expressed as:

$$S_M^*(f) = \frac{f S_M(f)}{(0.5\rho U_H^2 B H^2)^2} = \frac{S_p \beta (n/f_p)^\alpha}{\left\{1-(n/f_p)^2\right\}^2 + \beta (n/f_p)^2} \tag{2}$$

where B , H are width and height of the building respectively, $n = fB/U_H$, U_H is the mean wind speed at the top of the building with a 10-year return period ; f_p is the location parameter, deciding the peak frequency of the spectrum; β is the band width parameter; S_p is the amplitude parameter; and α is the deflection parameter. All the four parameters, which are functions of the aspect ratio, height ratio and wind field condition, can be identified by curve fitting technique[4].

The displacements ϕ of the story under modal conditions means the mode shape vector normalized with respect to the mass matrix[5]. Thus, acceleration response of the top floor can be simplified as

$$\sigma_{a(H)} = \frac{\phi_1^2(H)}{\sum_{z=0}^{H} (m(z)\phi_1^2(z))} \sqrt{\frac{\pi f_1 S_{F_1^*}(f_1)}{4(\zeta_{s1} + \zeta_{a1})}} = \frac{\phi_1^2(H)}{4\sum_{z=0}^{H} (m(z)\phi_1^2(z))} \rho H U_H^2 B \sqrt{\frac{\pi S_M^*(f_1)}{\zeta_{s1} + \zeta_{a1}}} \tag{3}$$

where $m(z)$ means the lumped mass of the story at the elevation z , ρ means the air density.

4.2. Modal Shape Updating Method

Through the wind-induced response analysis, we can obtain that the acceleration response of the top floor is determined by the period and modal shape of first vibration mode. In view of the natural vibration period is not quite effective when fine tuning sizes of structural members, the modal shape updating method is adopted to reduce the acceleration response by locally calibrating the modal shape near the floor where maximum building acceleration occurs. In each iteration of the modal shape updating process, the natural vibration period is assumed to remain unchanged. In this case, the constraints of acceleration response can be equally converted into the modal shape constraints.

$$a_H = c g_f \phi_1^2(H) \le a_H^l \Leftrightarrow \phi_1(H) \le \phi_1^U(H) \tag{4}$$

where g_f means the peak factor, c means the constant part in equation (3).

When the modal shape updates after an iteration, adverse impact of the natural vibration period on the acceleration response will be considered in the acceleration optimization process.

The mathematical model of modal shape updating method is introduced as follows:

$$\text{Minimize} \quad V = \sum_{j=1}^{m} A_j \sum_{jk=1}^{jn} l_{jk}$$

subject to

$$\phi = \frac{L^{(m)}}{6Q_s} \sum_{m=1}^{N} \left([\mathbf{F}_i^{(m)} \ \mathbf{F}_j^{(m)}] \begin{bmatrix} 2\mathbf{C}^{(m)} & \mathbf{C}^{(m)} \\ \mathbf{C}^{(m)} & 2\mathbf{C}^{(m)} \end{bmatrix} \begin{bmatrix} [\mathbf{f}_i^{(m)}]^T \\ [\mathbf{f}_j^{(m)}]^T \end{bmatrix} \right) \le \phi^l$$

$$0.6m \le b_j、\ h_j \le 1.5m \qquad (\mathrm{j}=1,2...\mathrm{m})$$
$$0.06m \le t_j \le 0.15m \qquad (\mathrm{j}=1,2...\mathrm{m})$$

where V is the volume of the material consumption (as the objective function in the optimization process), ϕ and ϕ^l are the modal shape of the initial structure and the modal shape limitation (this inequality as the constraint in the optimization process), $L^{(m)}$ means the length of m_{th} component, Q_s means the virtual load acting on the top floor, $F_i^{(m)}$ and $F_j^{(m)}$ are forces of the m_{th} component due to the modal load conditions while $f_i^{(m)}$ and $f_j^{(m)}$ for forces due to virtual load conditions, the matrix $C^{(m)}$ means the diagonal matrix of the m_{th} component where the elements are $1/EA$, $1/GA_Y$, $1/GA_Z$, $1/GI_X$, $1/EI_Y$ and $1/EI_Z$ respectively, b_j, h_j and t_j are the width, height and thickness of the flange and web of the steel members (as the design variables in the optimization process) .

By the modal shape updating method, the optimal section sizes for economical design of super tall buildings subject to wind-induced acceleration design requirements are obtained effectively.

5. Case Study

A real super tall building project is shown in this part to show the effectiveness of the effectiveness and applicability of the proposed modal shape updating method. The structure parameters and wind environment parameters are shown in Table 1 and Table 2.

Table 1: Structure parameters

H/m	B/m	D/m	f_1/Hz	$\zeta_s/\%$
598	68	68	0.107	1.5

Table 2： Wind field parameters

Wind field	$\omega_0/(kN/m^2)$	α	H_T	$\omega_H/(kN/m^2)$	$U_H/(m/s)$
C	0.30	0.22	400	0.90	37.3

The structural material is a mixed use of structural steel and reinforced concrete. It includes a central reinforced concrete core wall, eight exterior composite mega-columns in the perimeter and four at the corner. The central core walls are connected to the perimeter mega-columns by 5 outrigger trusses in X direction and 4 outrigger trusses in Y direction. The eight perimeter mega-columns and the 4 corner mega-columns are connected by nine belt trusses. The structural system and layout of outriggers are shown in Figure 1.

Outriggers are selected as the optimization design variables due to the fact that outriggers are the most sensible per volume for the modal updating according to the sensitivity analysis results. The outrigger members are divided into 10 groups, namely group 1~5 for the diagonal members and group 6~10 for the flange members in zone 2,4,6,7,8 respectively. The initial member sizes of diagonal members and flange members are $1.1m \times 1.1m \times 0.1m \times 0.1m$ and $1m \times 1m \times 0.1m \times 0.1m$ respectively. The lower and upper boundary of the member sizes are $0.6m \sim 1.5m$ for width and height and $0.06m \sim 0.15m$ for thickness.

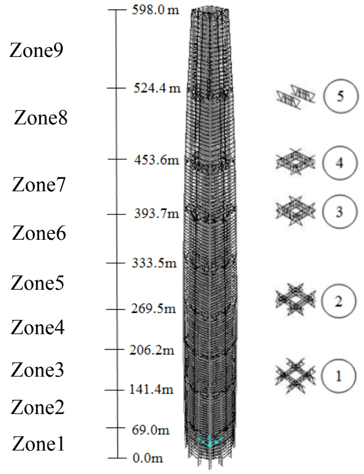

Figure 1: Structural system and layout of outriggers

According to the parameters in Table 1and Table 2, the relationship between non-dimensional power spectra of across-wind loads and the reduced frequency as shown in Figure 2. The power spectrum value will stay constant in the modal shape updating process as the frequency is assumed to be unchanged.

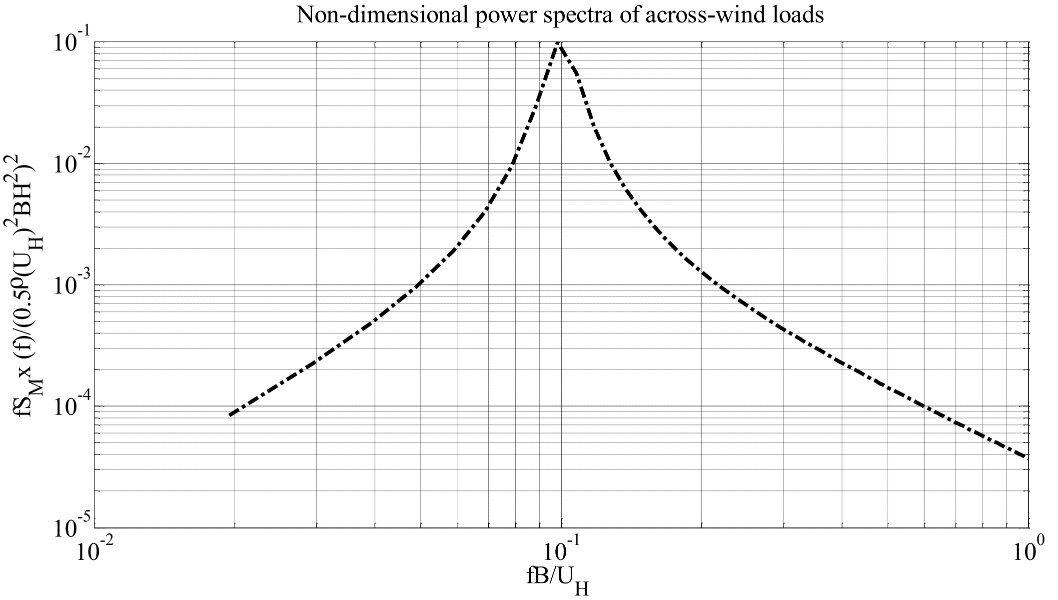

Figure 2: Non-dimensional power spectra of across-wind loads

The acceleration response of the top floor is 0.26 m/s^2 (4% over the 0.25 m/s^2 limitation in Chinese code). From equation (4), we can see clearly that the square of modal shape normalized with respect to the mass matrix value need to reduce by 4%, namely from 0.104 to 0.102, in the acceleration optimization process when the building period is assumed to be unchanged. When the modal shape updates after an iteration, the adverse impact of the natural vibration period on the acceleration response is taken into consideration to revise the modal shape variation range.

The member sizes of the outriggers before and after optimization are compared in Table 3.

Table 3: Comparison of member sizes of the outriggers before and after optimization

Members		α	h/m	t/m
Diagonal chords	Opti-1	1.1（0.83）	1.1（1.5）	0.1（0.06）
	Opti-2	1.1（1.2）	1.1（1.5）	0.1（0.06）
	Opti-3	1.1（1.355）	1.1（1.5）	0.1（0.06）
	Opti-4	1.1（1.5）	1.1（1.5）	0.1（0.15）
	Opti-5	1.1（1.5）	1.1（1.5）	0.1（0.15）
Flange chords	Opti-6	1（0.6）	1（0.6）	0.1（0.06）
	Opti-7	1（0.6）	1（1.5）	0.1（0.06）
	Opti-8	1（1.23）	1（1.22）	0.1（0.124）
	Opti-9	1（1.36）	1（1.42）	0.1（0.136）
	Opti-10	1（1.5）	1（1.5）	0.1（0.15）

From Table 3, we can come to the conclusion that heights of diagonal chords all increase to the upper boundary $1.5m$ while width of diagonal chords only in zone 7 and 8 comes to the upper boundary, increase a little in zone 2, decreased in zone 1. Heights and widths of the flange chords in zone 6, 7, and 8 all increased a lot. Flange chords in zone 2 and 4 decreased to the lower boundary. The outrigger members in higher zones and diagonal chords contribute more for the modal shape normalized with respect to the mass matrix of the top floor.

Figure 3 and Figure 4 compare the modal shape normalized with respect to the mass matrix and normalized with respect to the modal amplitude of the top floor before and after optimization. We can obtain that the modal shape normalized with respect to the mass matrix reduce to 0.1012 from Figure 3. Figure 4 shows that the modal shape normalized with respect to the modal amplitude of the top floor after optimization is mostly larger than the initial mode shape. The results indicate that the modal shape updating method brings a larger generalized mass of the structure from another perspective.

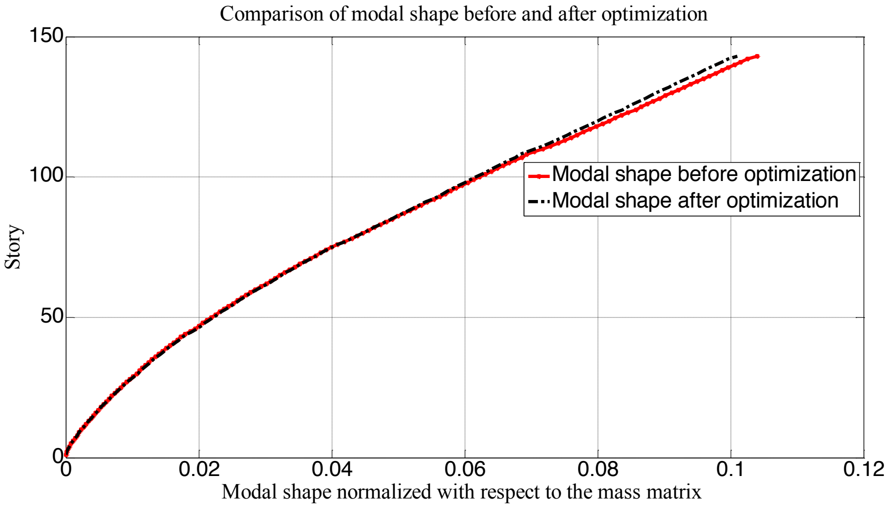

Figure 3: Comparison of modal shape normalized with respect to the mass matrix

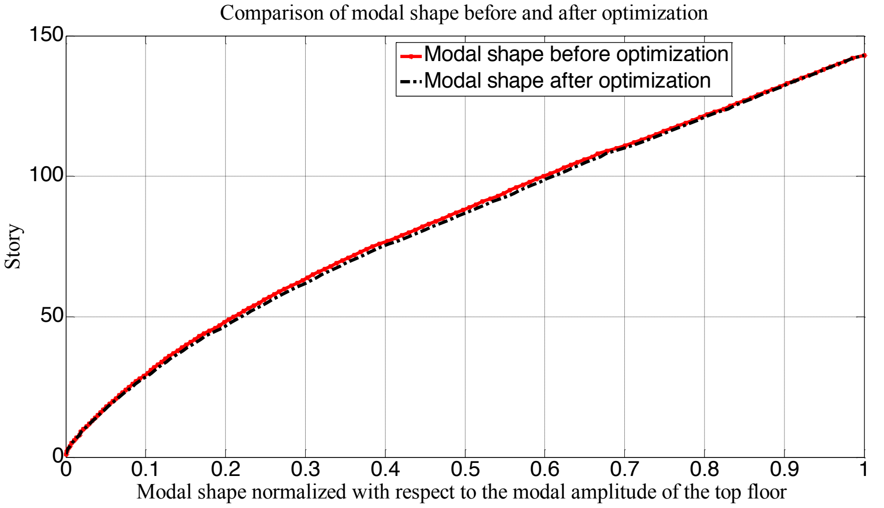

Figure 4: Comparison of modal shape normalized with respect to the modal amplitude of the top floor

Change of modal shape normalized with respect to the mass matrix is shown in Figure 5. As we can see from equation (4), the optimization of modal shape (reducing by 2.7%) would bring a 5.3% reduction in the acceleration response when the building period is assumed to be unchanged. Since the acceleration exceeds the code limit by only 4%, the extra optimized 1.3% was for the compensation of the longer building period (from 9.29s to 9.36s).

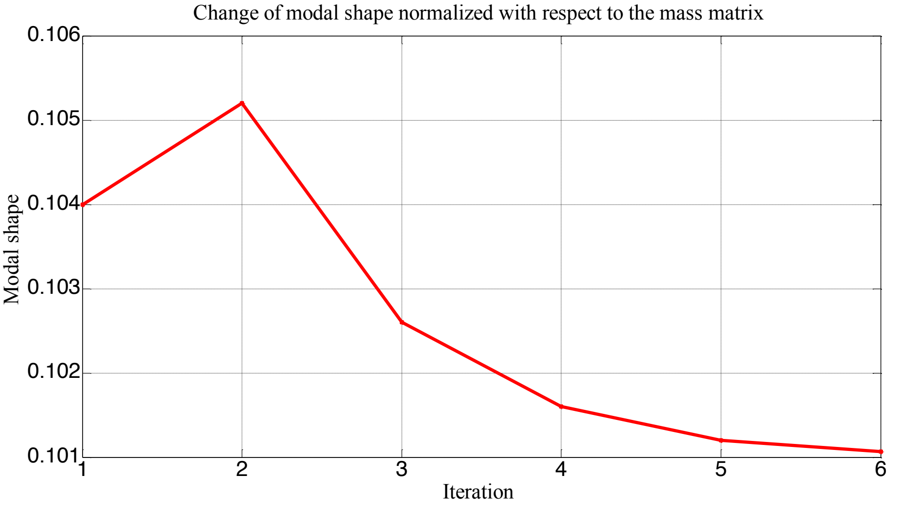

Figure 5 Change of modal shape normalized with respect to the mass matrix

Change of volume of outrigger members is shown in Figure 6. 15% (54m^3) additional steel is needed for the acceleration optimization by the modal shape updating method.

Figure 6 Change of volume of outrigger members

6. Conclusion

The modal shape updating method provides a powerful tool for wind-induced dynamic serviceability design of super tall building structures with slightly excessive wind-induced acceleration responses. Through the real super tall building project to show the effectiveness and applicability of the proposed modal shape updating method, we can come to the conclusion that:

(1) Outriggers are selected as the optimization design variables due to the fact that outriggers are the most sensible per volume for the modal updating. The outrigger members in higher zones and diagonal chords contribute more for the modal shape normalized with respect to the mass matrix of the top floor;

(2) The optimization of modal shape normalized with respect to the mass matrix (reducing by 2.7%) would bring a 5.3% reduction in the acceleration response when the building period is assumed to be unchanged. Since the acceleration exceeds the code limit by only 4%, the extra optimized 1.3% was for the compensation of the longer building period (from 9.29s to 9.36s);

(3) The modal shape normalized with respect to the modal amplitude of the top floor after optimization is mostly larger than the initial mode shape, which indicates that the modal shape updating method brings a larger generalized mass of the structure from another perspective.

(4) 15% (54m^3) additional steel is needed for the acceleration optimization. The expense for the acceleration optimization is acceptable.

5. Acknowledgements

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