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Novel Approach in Topology Optimization of Porous Plate Structures for Phononic Bandgaps of Flexural Waves

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1. Abstract
This paper presents a novel multiobjective optimization strategy for topology optimization of single material phononic plates (PhPs), where the achieved topology can be produced by perforation of a uniform background plate. The primary objective of this optimization study is to exploit the widest relative bandgap of fundamental flexural guided wave modes for maximized phononic controllability. Principally, the optimum topology of such porous structure favors isolated scattering domains leading to maximized interfacial Bragg reflections. Hence, the widest achievable bandgap depends on assumed topology resolution and relevant topology generally has low structural worthiness. Therefore the homogenized in-plane stiffness of phononic unitcell is also incorporated in topology optimization as the second objective to explore the gradient of optimum bandgap topology with respect to its in-plane stiffness. Consequently, structurally worthy bandgap topologies with desired relative bandgap-stiffness performance could be taken from the obtained spread of optimized topologies. Moreover, functionally graded PhP with maximized bandgap efficiency and multiscale functionality could be designed through integration of optimized PhP unitcells of different stiffness. Nondominated sorting genetic algorithm (NSGA-II) is adopted for this multiobjective problem and fitness evaluation of topologies is performed through finite element method. Specific topology assessment is performed for convergence of the solution towards optimum feasible bandgap topology without penalizing the efficiency of genetic algorithm (GA). A set of Pareto topologies is selected and variation of bandgap width and in-plane stiffness across the two Pareto extremes is studied. Arbitrarily selected intermediate Pareto topology shows superior bandgap efficiency as compared with the relevant optimized topologies reported by other researchers. Moreover, the frequency response of a finite phononic plate structure of selected intermediate Pareto topology confirms high attenuation of flexural waves within its calculated bandgap frequency.
2. Keywords: Phononic; Bandgap; Plate; Topology Optimization

3. Introduction

Phononic crystals (PhCr) are heterogeneous acoustic meta-materials produced by periodic modulation of acoustic impedance in a lattice structure through either integration of two or more contrasting materials, or making porosities in a uniform background. The main feature of PhCrs is the existence of frequency ranges over which propagation of vibroacoustic waves is prohibited. This phenomenon is caused by constructive reflection and superposition of waves at the interface of periodic heterogeneities i.e. Bragg and Mie resonant scatterings. Moreover, introducing any kind of defect in phononic lattice e.g. by altering the features of a few adjacent cells, leads to advent of local resonance modes within phononic bandgap frequency. This capability is used to trap and guide waves inside defects specially tuned for desired frequencies. Flat and concave wave fronts below bandgap frequency are other promising characteristics of PhCrs applicable for self-collimation and steering of waves. Consequently, it is worthwhile to adjust the phononic properties so that the width and location of bandgap is optimized for application of interest. Essentially topology with maximum relative bandgap width (RBW) between subsequent modes of interest is desired [1]. RBW is the ratio of bandgap width over midgap frequency, and so seeking maximum RBW results in a topology which can provide widest bandgap at lowest frequency range for specific unitcell size. Consequently, the relevant topology supports phononic wave manipulation over the widest frequency range through miniature unitcells compared to wavelength.

The efficiency of PhCr is principally defined by the topology of its irreducible representative element (Unitcell). Most of topology optimization studies in relation to 2D periodic PhCrs have been concerned with bandgaps of in-plane and/or anti-plane bulk waves while only few works have been devoted to bandgaps of guided waves through PhCr plate (PhP). Guided waves are structural waves confined by traction free surfaces of thin walled structures, so called plate waves when guided by parallel faces of plates. In-plane symmetric (S) and anti-plane asymmetric (A) Lamb waves as well as symmetric shear horizontal (SHS) and asymmetric shear horizontal (SHA) in-plane waves are the well-known guided wave modes generated in a plate structure. The plate wave energy is predominantly conveyed by the asymmetric (flexural) wave modes (S+SHA) under a general lateral disturbance. Special characteristics of guided waves confined by finite thickness of such structures, make them ideal for non-destructive evaluation purposes [2] as well as production of low loss resonators, filters and waveguides [3].

Halkjær, Sigmund [4] studied the optimum topology of porous Polycarbonate PhP with rhombic unitcell for maximized RBW of first couple of flexural plate waves. The Mindlin plate theory was implemented for definition of band structure of bending waves and gradient based optimization was performed through method of moving asymptotes. The discontinuities of optimized topology were locally modified for satisfactory stiffness and manufacturability. In another investigation by Bilal and Hussein [5] the optimum topology of thin porous silicon PhPs for maximized RBW of basic flexural waves was studied. The Mindlin plate's theory was implemented and topology of square unitcell was optimized through genetic algorithm (GA) as an evolutionary based method.

Bandgaps of such porous materials are governed by wave reflection and scattering at the interface of inhomogeneities produced by perforation profile. Therefore the search for highest RBW naturally results in topologies with nearly isolated domains or in other words thin connectivity for strengthened interfacial reflections. The finer the topology resolution the thinner connectivity in the optimized topology for maximized bandgap. So largest achievable RBW is extremely dependent on assumed unitcell's resolution and relevant topology generally has low structural worthiness. Nevertheless, none of earlier works on topology optimization of porous PhCrs [4-6] took into consideration the structural worthiness of achieved topologies.

Therefore, the focus of this paper is to investigate the contribution of this fact in optimum topology of PhP unitcell with 2D periodicity for maximized bandgap width of fundamental flexural waves (A_0+SHA_0). To serve this purpose, the homogenized in-plane stiffness of phononic unitcell is also incorporated in topology optimization as the second objective and structurally worthy bandgap topologies are explored. The gradient of optimum bandgap topology with respect to in-plane stiffness is also defined by which functionally graded PhP with maximized bandgap efficiency and multiscale functionality can be designed. Nondominated sorting genetic algorithm (NSGA-II) is adopted for this multiobjective problem and fitness evaluation of topologies is performed through finite element method. Relatively thin phononic unitcell of transversal aspect ratio (width to thickness) of 10 is modelled and square symmetric topology with no through thickness variation is assumed. Specialized filtering is applied to the topologies in order to incline the search space towards feasible bandgap topologies without compromising its diversity and randomness.

4. Modelling and analysis of PhP unitcell

2D PhP unitcell with square symmetry is assumed which is uniformly perforated through the thickness h (along z axis) to produce planar heterogeneity in x-y plane. The topology (i.e. perforation profile) of this porous PhP defines its bandgap efficiency caused by constructive reflections of wave at the interface of heterogeneities. Relatively thin plate unitcell with transversal aspect ratio (width over thickness) of $a/h=10$ is considered. 3D FEM model of the unitcell is constructed in ANSYS APDL (*ANSYS® Academic Research, Release 14.5)* using SOLSH190 elements, for fitness evaluation of individual topologies during optimization procedure. Polysilicon with elastic modulus $E_s=169(GPa)$, Poisson's ratio $\upsilon_s=0.22$ and density $\rho_s=2330(kg/m^3)$ is taken as the base solid material of PhP widely used for fabrication of micro-devices like micro PhCrs [7].

4.1. Modal band analysis and bandgap objective

The FEM notation of equation of motion for a loss free elastic medium in the absence of external forces is:

$$\mathbf{M\ddot{q}}+\mathbf{Kq}=0 \tag{1}$$

where $\mathbf{K}$ is stiffness matrix and $\mathbf{M}$ is mass matrix of FEM model. $\mathbf{q}$ is vector of nodal displacements by which the displacement vector $\mathbf{U}=\{u,v,w\}$ is defined through matrix of shape functions $\mathbf{N}$:

$$\mathbf{U}=\mathbf{Nq} \tag{2}$$

Based on the Bloch-Floquet theory[8] the general solution for displacement filed $\mathbf{U}$ in 2D PhP with infinite periodicity in x-y plane can be formulated by harmonic modulation of a periodic displacement filed as follows:

$$\mathbf{U}(\mathbf{X},t)=\mathbf{U_p}(\mathbf{X})e^{i(\mathbf{kx}-\omega t)} \tag{3}$$

where $\mathbf{X}=\{x,y,z\}$ is the location vector, $\mathbf{U_P}$ is an a periodic function in x-y plane according to the PhP square lattice periodicity, $\omega=2\pi f$ is circular frequency and $\mathbf{k}=\{k_x,k_y\}$ is the in-plane wave vector and $i=\sqrt{-1}$. After applying Bloch-Floquet periodic boundary condition on the plate unitcell with traction free upper and lower surfaces, we arrive at:

$$\mathbf{U_{X+}}=\mathbf{U_{X-}}e^{ik_xa} \quad , \quad \mathbf{U_{y+}}=\mathbf{U_{y-}}e^{ik_ya} \tag{4}$$

where subscripts x+ and x- stand for corresponding points at two unitcell's faces parallel to y-z plane at $x = 0$ and $x = a$, respectively. Similarly subscripts y+ and y- stand for corresponding points at two unitcell's faces parallel to z-x plane at $y = 0$ and $y = a$. So the Bloch-Floquet boundary condition is in complex format and in order to handle it by ANSYS FEM solver two identical FEM models are superimposed so that one represents the real term and the other one imaginary term of complex displacement domain [9]. Due to the periodicity in Bloch-Floquet condition and the assumed square symmetry of topology the wave vector can be limited to irreducible Brillouin zone. However, according to the common practice searching only the border of irreducible Brillouin zone [1] suffices for evaluation of bandgap properties. By substituting the harmonic definition of nodal displacement $\mathbf{q} = \mathbf{q}_0 e^{i\omega t}$ in Eq.(1) and solving for nontrivial solutions, the modal frequencies of unitcell are calculated by Eigen value analysis of Eq.(5):

$$\mathbf{M}\omega^2 + \mathbf{K}(\mathbf{k}) = 0 \tag{5}$$

where stiffness matrix $\mathbf{K}(\mathbf{k})$ is a function of wave vector $\mathbf{k}$ corresponding to applied Bloch-Floquet boundary conditions. In order to decouple the modal band structure of flexural plate waves, just half of unitcell through the thickness from midplane $z = 0$ to top plane $z = h/2$ is modelled and appropriate boundary conditions are applied to the mid-plane to enforce asymmetric modes [10].

Regarding bandgap efficiency of PhP unitcell, it is fundamentally desired to manipulate the largest wave length possible through specific unitcell size. Hence the first objective function of optimization F_1 to be maximized is defined as:

$$F_1 = \frac{\min_{i=1}^{n_k} \omega_{j+1}^2(k_i) - \max_{i=1}^{n_k} \omega_j^2(k_i)}{0.5(\min_{i=1}^{n_k} \omega_{j+1}^2(k_i) + \max_{i=1}^{n_k} \omega_j^2(k_i))} \tag{6}$$

Actually F_1 is the gap width over mean value of gap between Eigen values ω^2 of two subsequent modal branches j^{th} and $(j+1)^{th}$ of interest over the n_k discrete search points k_i on the border of irreducible Brillouin zone. Since the bandgap of fundamental flexural modes is desired, the first couple of modal branches are taken into account and so $j = 1$.

4.2. Numerical homogenization and stiffness objective

The strain energy compliance of PhP unitcell under particular loading condition is defined as the second objective function of optimization concerning the in-plane stiffness. It is well known that the strain energy of the unitcell under specified loading condition represents its relevant stiffness. Higher strain energy implies lower stiffness or higher compliance. The strain energy stored per volume of a finite element subjected to in-plane stress in x-y plane is theoretically defined as:

$$S = \frac{1}{2}(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy}) \tag{7}$$

where $\{\sigma_x, \sigma_y, \tau_{xy}\}$ and $\{\varepsilon_x, \varepsilon_y, \gamma_{xy}\}$ are corresponding in-plane stress and strain tensors, respectively. As for loading conditions the general stress state of $\sigma_x = \sigma_y = \tau_{xy}/\sqrt{2} = \sigma$ with shear and biaxial normal components is assumed uniformly throughout the unitcell. So with regard to linear elastic stress-strain relations, the Eq.(7) turns to:

$$S = \sigma^2 (\frac{1 - \nu_e}{E_e} + \frac{1}{G_e}) \tag{8}$$

where E_e, G_e and υ_e are the equivalent in-plane orthotropic elastic modulus, shear modulus and Poisson's ratio of assumed square symmetric PhP unitcell, respectively. Then the stiffness objective function F_2, in order to attain maximum in-plane stiffness, is defined as strain energy compliance of unitcell based on its homogenized properties, to be minimized:

$$F_2 = \left(\frac{1 - \nu_e}{E_e}\right) + \left(\frac{1}{G_e}\right) \tag{9}$$

So the assumed relative magnitude of shear and normal stresses leads to an stiffness objective which equally accounts for normal and shear compliances (first and second terms of Eq.(9), respectively). However appropriate stress state could be assigned for any particular application .The numerical homogenization is employed and periodic boundary condition is applied to define the equivalent elastic properties of PhP unitcell and calculate F_2 .

An overall test strain is prescribed for the unitcell and the equivalent orthotropic stiffness is derived based on average stress filed of elements over the entire unitcell. Detailed information regarding numerical homogenization of composite materials using FEM and periodic boundary condition can be found in Bendsoe and Sigmund [11].

5. Optimization approach

Non-dominated sorting Genetic algorithm (NSGA-II) [12] is employed for present multiobjective topology optimization problem. Basically a coded string of topology is processed by the algorithm to be mapped into the discretized design domain. Genetic algorithm (GA) mimics the process of natural selection: searches over a population of potential solutions, reproduces better generations using crossover and mutation functions and iteratively produces better and better approximations to the solution. Earlier works on topology optimization of PhCrs through GA have proved its competency in this area [5, 13-19]. Multi-objective GA was implemented by the authors to study the optimum topology of 1D PhPs with respect to filling fraction of constituents and gradient 1D PhP structures were introduced [20]. NSGA-II sorts GA population based on non-dominated fitness and crowding distance. In this way the designs with the most dominated fitness are placed at the first rank (Pareto front) and among them those with highest crowding distance are preferred [12].

A binary bit-string design space is considered for defining the material type of any pixel of topology. For simplicity in FEM evaluation of topologies, the void elements corresponding to perforated region of unitcell are assigned a very compliant material. Following specialized topology initializing, filtering and modifications are applied to eliminate interrupting modes of disconnected segments and make optimized porous topologies converge towards feasible connected ones without disturbing the randomness and diversity of search domain. An identical fully solid population is randomly mutated with specified probability to deliver an initial population of likely connected topologies. All individual topologies are morphologically checked and the feasible connected segment only (if any) is delivered for analysis. The population is sorted and parent pool is created through tournament selection. Then the offspring population is defined by single point crossover and mutated with certain probability. The checkerboard problem is alleviated by the well-known technique of inversing randomly selected pixel (i.e. void to solid and vice versa) if surrounded by more than 2 dissimilar adjacent face to face pixels out of 4. Finally the idle disconnected segments of topology (where applicable) are faded out through a random-progressive manner. For this purpose a randomly selected disconnected segment is eliminated by specified probability which increases as the GA progresses. Due to rarity of perfectly connected topologies showing bandgap properties defined during population initialization and reproduction stages, the aforementioned algorithm is a robust approach for degrading invalid individuals while maintaining the diversity of search domain.

6. Results and discussion

PhP unitcell is modelled by element (i.e. topology) resolution of 32×32 in x-y plane leading to a design domain of 136 independent design variables for prescribed square symmetric topology. A population size of 200 is considered for this problem size, and from the total number of topologies delivered in Pareto front 12 topologies (Figure 1(a)) are selected and presented containing a variety of alternative optimized topology modes with different RBWs and in-plane stiffness.

Since RBW is defined as the gap width over midgap value of either Eigen values (F_1) or Eigen frequencies ($\Delta\omega / \overline{\omega}$) of desired mode branches in different works both quantities are obtained and included in the results. In order to evaluate the relative effective stiffness of optimized topologies with respect to stiffness of constitutive solid material, relative elastic modulus $E_r = E_e / E_s$ and relative shear modulus $G_r = G_e / G_s$ are calculated.

The gradient of RBW (both F_1 and $\Delta\omega / \overline{\omega}$) and logarithm of relative elastic modulus with respect to selected Pareto front topologies are given in Figure 1(b) and Figure 1(c), respectively. Accordingly, almost uniform reduction of RBW across the Pareto front is observed with simultaneous increase of both elastic and shear modulus by around 2 orders of magnitude. As expected, the stiffness of PhP unitcell penalizes its bandgap efficiency and the bandgap of extreme topology number 12 with highest stiffness is almost closed. Hence, the choice of optimum Pareto topology depends on desired relative bandgap-stiffness performance. In order to compare the performance of obtained topologies with preceding works on bandgaps of asymmetric waves, the intermediate Pareto topology number 6 (Figure 1(a)) is arbitrarily chosen and remodeled with Polycarbonate solid material of elastic modulus 2.3 GPa, density 1200 kg/m^3 and Poisson's ratio 0.35 and aspect ratio $r = 11$ as per work by Halkjær, Sigmund [4]. Accordingly the relevant frequency RBW of this topology is $\Delta\omega / \overline{\omega} = 0.328$ which is significantly higher than that of optimized connected topology reported by Halkjær, Sigmund [4] with $\Delta\omega / \overline{\omega} = 0.16$. Furthermore, the Eigen value RBW of topology number 6 remodeled with Polycarbonate solid material and $r = 11$ is $F_1 = 0.639$ which is around 13% higher than $F_1 = 0.5639$ reported for optimized topology obtained by Bilal and Hussein [5].

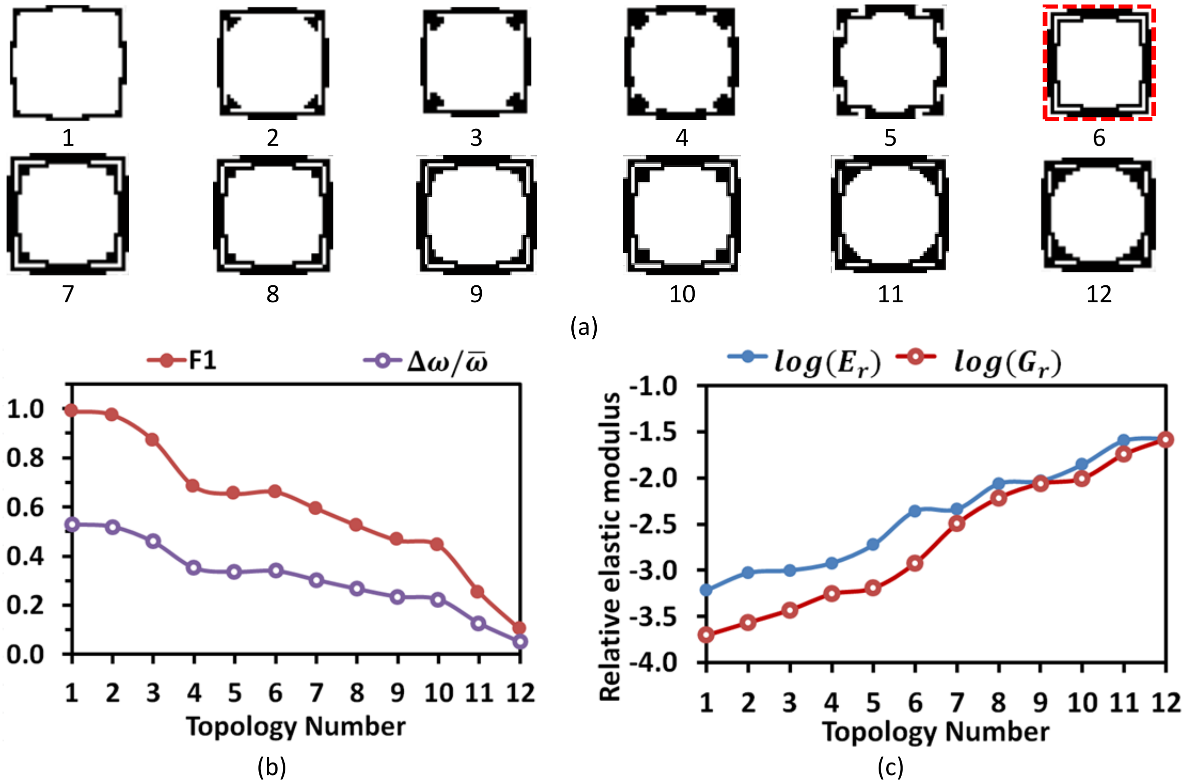

Figure 1: (a) Selected optimized topologies, and gradient of (b) RBW and (c) elastic properties across Pareto front

It is worth to emphasize that bandgap performance of an intermediate Pareto topology (Number 6), and even not the one with widest RBW (Number 1), was compared with results of other works aimed at topology with absolutely maximized RBW. Moreover, the topologies of present study are optimized based on marginally lower aspect ratio $r = 10 < 11$.

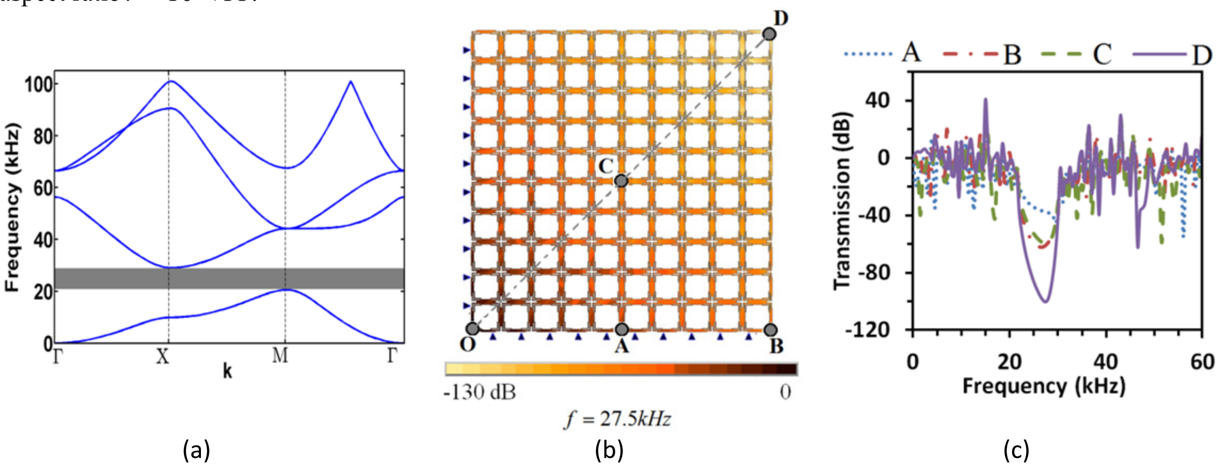

Figure 2: (a) Modal band structure of topology number 6 with 1 mm thickness, (a) harmonic response of its finite square PhP structure at bandgap frequency 27.5 kHz and (c) transmittance to selected measurement points

Finally the steady state harmonic response of a finite PhP structure of selected topology number 6 is defined in order to evaluate its bandgap performance compared with its modal band structure calculated for an infinitely periodic lattice. Unitcell of width $a = 10mm$ and thickness $h = 1mm$ is modeled and the corresponding modal band structure is shown in Figure 2(a). Accordingly, a bandgap is opened within first couple of flexural modes from just above 20 kHz to around 30 kHz. A square lattice of 20×20 unitcells with free boundaries is considered to be transversally loaded at the center for predominant excitation of asymmetric modes. Due to symmetry of the model, the top right quarter of plate is modelled only as shown in Figure 2(b). The plate is harmonically excited at central point O in the range 0-60 kHz corresponding to first two modal branches. Then the wave transmittance to the straight points A and B and diagonal ones C and D is measured as shown in Figure 2(c). It is evident that the transmittance of elastic wave is highly attenuated within bandgap frequency 20-30 kHz with highest damping of around -100 dB at the most distant point D in 27.5 kHz and lowest damping of -40dB at closest point A in 30 kHz. The contour of wave transmittance corresponding to the bandgap frequency 27.5 kHz as presented in Figure 2(b) confirms the omnidirectional attenuation of wave up to -130 dB throughout the modelled finite PhP structure.

7. Conclusion

Topology optimization of porous phononic plates for widest bandgap of fundamental flexural waves was competently performed through specialized genetic algorithm. Multiobjective study was carried out and as a result the variation of bandgap performance with respect to effective in-plane stiffness of optimized topologies was explored. Specific topology processing was implemented to get feasible porous topologies without compromising the efficiency of genetic algorithm and diversity of design space. A broad range of Pareto topologies was delivered enabling the designer to select supreme topology based on required bandgap-stiffness performance. Arbitrarily chosen intermediate Pareto topology had excellent bandgap efficiency as compared with relevant results of other researchers, and omnidirectional attenuation of flexural waves through its finite square structure within calculated bandgap frequency was computationally observed.

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