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Achieving Minimum Length Scale in Topology Optimization by Geometric Constraints

Mingdong Zhou[1,2], Boyan S. Lazarov[1], Fengwen Wang[1], Ole Sigmund[1]

[1] Dept. of Mechanical Engineering, Technical University of Denmark, Kgs. Lyngby, Denmark, minzho@mek.dtu.dk
[2] Dassault Systèmes Deutschland GmbH, Simulia Office, Karlsruhe, Germany

1. Abstract
A topology optimization approach is presented to design structures with strict minimum length scale. The idea is inspired from the work on topology optimization with robust formulations [1, 2, 3], where the optimized nominal design possess minimum length scale if all the considered design realizations in the problem formulation share a consistent topology. However, the latter condition may not be satisfied depending on physical problems [2]. In the current study, two differentiable geometric constraints are formulated based on a filtering-threshold modeling scheme. Satisfying the constraints leads to designs with controllable minimum length scale on both solid and void phases. No additional finite element analysis is required for the constrained problem. Conventional topology optimization can be easily extended to impose minimum length scale on the final design with the proposed constraints. Numerical examples of designing a compliant mechanism and a slow-light waveguide are presented to show the effectiveness of this approach.

2. Keywords: topology optimization, minimum length scale, geometric constraint

3. Introduction
There has been tremendous interest in prototyping topologically optimized designs without the tedious post-processing in CAD softwares, thanks to the boosting macro- / micro- 3D printing technology. On the side of topology optimization, the optimized design must contain no single structural member whose size is below the resolution of a 3D printer in order to avoid prototype deficiency, such as holes or disconnected parts. One solution is to achieve minimum length scale in the topology optimization results. Another benefit of doing so is that, if a compliant mechanism is designed achieving minimum length scale helps guarantee a longer device life-time by preventing tiny-hinges appearing at structural joints (high-stress regions).

Previous attempts have been made to impose minimum length scale in topology optimization. Within the density-based approach [4], Poulsen proposed the so-called MOLE (MOnotonicity based minimum LEngth scale) method [5], which achieves minimum length scale larger than the size of a circular "looking glass". Guest [6] suggested projection schemes by projecting the nodal density into the element space of a minimum feature size. However, it does not resolve the "one-node hinge" problem in designing compliant mechanisms and simple projection may result in grey scale for some design problems [1]. The robust formulations [1, 2, 3], which take the eroded, dilated and (one or several) intermediate design realizations into account at the same time, impose length scale on the intermediate blueprint design only if the considered design realizations share the same topology. However, as pointed out in [2], the robust formulation does not necessarily guarantee a consistent topology for the realizations in different physical problems and the length scale can only be checked a posteriori. Another drawback of using a robust formulation is the high computational cost, that finite analysis is basically required for each design realization in every iteration. A perturbation based technique [7] proposed by Lazarov et. al is a computationally efficient solution. However, due to the locality of the approximation, it cannot provide a clear length scale control for compliant mechanism problems. Recently, a skeleton-based idea, which is similar to that in [8] with the level set method, is implemented using a density based method in [9]. Both minimum and maximum length scale are considered in their works [8, 9]. However, the sensitivity regarding the change of the medial-zone is neglected in the sensitivity analysis. Possible shortcomings of this approach are discussed in detail in [10]. Within the level set based method [11], Chen et al. [13] applies a quadratical energy functional to design a thin elongated structural layout with length scale. However, there is no explicit way to define the exact length scale with this formulation. A rigorous mathematic approach for imposing minimum and maximum length scale in level set based topology optimization is proposed in [10, 12]. Besides the above approaches, predefined engineering features with length scale can be designed and optimized using a CSG based level set approach as discussed in [14]. Another option is to directly consider the manufacturing characteristics in the optimization process, which can always ensure manufacturable designs of optimized performance [15, 16].

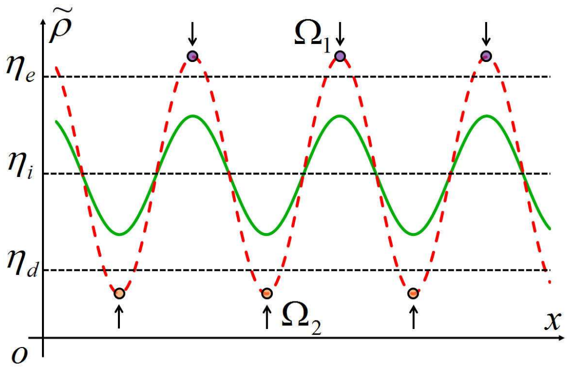


Figure 1: Re-designing a filtered field (from the solid to the dashed curve) to satisfy the conditions $(i)-(ii)$ according to Eqs. (1) and (2). Minimum length scale is achieved on the physical field thresholded by η_i for the dashed curve.

In this study, a new approach is proposed based on a filtering-threshold topology optimization scheme [17], which utilizes a design field ρ ($0 \leq \rho \leq 1$), a filtered design field $\tilde{\rho}$ and a projected (physical) field $\bar{\rho}$. The idea is inspired from the work on topology optimization with robust formulations [1, 2, 3], where the optimized nominal design possess minimum length scale if all the considered design realizations $\bar{\rho}_\eta$ thresholded in a range $\eta \in (\eta_d, \eta_e)$ ($0 < \eta_d < \eta_i < \eta_e < 1$) share a consistent topology. One sufficient condition to the latter is as follows:

$$(i) \quad \tilde{\rho}(\mathbf{x}) \geq \eta_e, \quad \forall \mathbf{x} \in \Omega_1 = \{\mathbf{x} | \bar{\rho}_{\eta_i}(\mathbf{x}) = 1 \text{ and } \nabla\tilde{\rho} = 0\}; \tag{1}$$

$$(ii) \quad \tilde{\rho}(\mathbf{x}) \leq \eta_d, \quad \forall \mathbf{x} \in \Omega_2 = \{\mathbf{x} | \bar{\rho}_{\eta_i}(\mathbf{x}) = 0 \text{ and } \nabla\tilde{\rho} = 0\}; \tag{2}$$

where Ω_1 and Ω_2 represent the inflection region of the filtered field in the solid and void phase of the physical field, respectively. Fig. 1 illustrates this idea with a 1D example. The solid curve represents an initial filtered field, for which the physical fields thresholded at η_d, η_i, η_e possess different topologies. Re-designing it into the dashed curve as shown in the figure to satisfies (i) and (ii), a solid phase is ensured in Ω_1 for all the physical realizations thresholded by $\eta < \eta_e$ and a void phase remains in Ω_2 within $\eta > \eta_d$. As a result, all the physical realizations thresholded by $\eta \in (\eta_d, \eta_e)$ share a consistent topology and minimum length scale is expected on the blueprint design $\bar{\rho}_{\eta_i}$ for both solid and void phases. In this work, two geometric constraints are proposed to meet the above condition. By solving a constrained optimization problem, minimum length scale is realized without additional finite analysis, which alleviate the computational burden of using a robust formulation. This approach is purely geometrical and problem independent. Existing topology optimization framework can be easily extended to have minimum length scale in final optimized design by incorporating the proposed geometric constraints.

4. Geometric constraints for minimum length scale

The filtering-threshold topology optimization scheme [17] consists of a design field ρ, a filtered field $\tilde{\rho}$ and a physical field $\bar{\rho}$, whose relations are defined as follows:

$$\tilde{\rho}_i = \frac{\sum_{j \in \mathbf{N}_i} \omega(\mathbf{x}_j) v_j \rho_j}{\sum_{j \in \mathbf{N}_i} \omega(\mathbf{x}_j) v_j}, \quad \omega(\mathbf{x}_j) = R - |\mathbf{x}_i - \mathbf{x}_j|, \tag{3}$$

$$\bar{\rho}_i = \frac{\tanh(\beta \cdot \eta) + \tanh(\beta \cdot (\tilde{\rho}_i - \eta))}{\tanh(\beta \cdot \eta) + \tanh(\beta \cdot (1.0 - \eta))}. \tag{4}$$

where $\mathbf{N}_i$ is the set of elements in the filter domain of the element i, R is the radius of the filter, v_j is the volume of the element j, β controls the steepness of the approximated Heaviside function and η is the threshold.

In order to fulfill the two requirements in Eqs. (1) and (2), two structural indicator functions are first defined to capture the inflection regions Ω_1, Ω_2 defined in Eqs. (1-2):

$$I^s = \bar{\rho} \cdot exp(-c \cdot |\nabla\tilde{\rho}|^2), \tag{5}$$

$$I^v = (1 - \bar{\rho}) \cdot exp(-c \cdot |\nabla\tilde{\rho}|^2), \tag{6}$$

where the subscripts s and v stand for the solid and void phase, respectively. The exponential term in Eqs. (5) and (6) annotates the inflection region of a filtered field ($|\nabla\tilde{\rho}| = 0$) with value 1, while the parameter c controls the decay rate of I^s and I^v wherever $|\nabla\tilde{\rho}| \neq 0$. Because of numerical errors in calculating the gradient value $(\nabla\tilde{\rho})$, the parameter cannot be set arbitrarily large. Numerical experience implies that setting $c = r^4$ (where $r = R/h$ and h represents the element size) is effective in practice in capturing the inflection region during the optimization process. Based on the indicator functions, the geometric constraints are proposed as:

$$g^s = \frac{1}{n}\sum_{i\in\mathbf{N}} I_i^s \cdot [\min\{\,(\tilde{\rho}_i - \eta_e)\,,\,0\,\}]^2 = 0, \tag{7}$$

$$g^v = \frac{1}{n}\sum_{i\in\mathbf{N}} I_i^v \cdot [\min\{\,(\eta_d - \tilde{\rho}_i)\,,\,0\,\}]^2 = 0, \tag{8}$$

where n is the total number of elements. Satisfying these two constraints results in the value of the filtered field being larger than the threshold η_e at the inflection region Ω_1 and smaller than the threshold η_d at Ω_2. Therefore, the sufficient condition is satisfied and minimal length scale is expected over the nominal design. The proposed geometric constraints are differentiable w.r.t. the design variable ρ and computationally cheap. They can be obtained from the value of the physical field $\bar{\rho}$, the filtered field $\tilde{\rho}$ and its gradient $\nabla\tilde{\rho}$, which are directly available during the optimization process.

In practice, the equality constraints Eqs. (7-8) cannot be strictly satisfied due to numerical errors. It is pertinent to apply an relaxed version to a topology optimization problem:

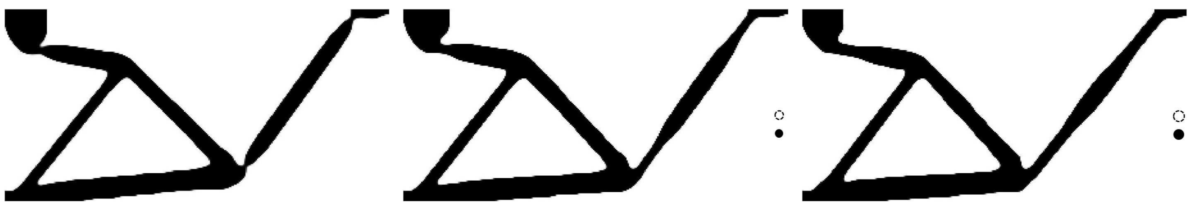
$$\begin{aligned} \min: &\quad F(\mathbf{u}(\boldsymbol{\rho}),\boldsymbol{\rho}), \\ \text{s.t.}: &\quad g_j \leq 0, \quad j = 1:m, \\ : &\quad g^s \leq \varepsilon, \\ : &\quad g^v \leq \varepsilon, \\ : &\quad 0 \leq \boldsymbol{\rho} \leq 1. \end{aligned} \tag{9}$$

where F and g_j are the objective functional and constraints of the original problem respectively and ε is a small number. The minimum length scale on the final result is determined by the radius of the filter, the considered threshold range (η_d, η_e) and the threshold η_i for the blueprint design. Readers are referred to [2, 18] for predicting the minimum length scale using the linear-hat based filtering.

5. Numerical Examples

In this section, the proposed approach is applied to two benchmark examples. The constrained problem (9) is solved by the method of moving asymptotes (MMA) [19].

The first example is to design a compliant inverter with minimum length scale. The optimization problem is to maximize the output displacement, of which the direction is opposite to that of the external force. The detailed problem formulation and parameter definitions can be found in [2]. Fig. 2(a) shows a standard topologically optimized inverter containing tiny hinges at structural joints. The tiny hinges will cause high stress when the mechanism deforms and thus they shall be prevented in the blueprint design. Fig. 2(b,c) shows two such improved designs by applying the proposed geometric constraints. Different minimum length scales are achieved by adopting different threshold ranges $(\eta_d, \eta_e) = (0.4, 0.6)$ and $(0.3, 0.7)$, respectively. As the minimum length scale increases, the deforming capability (the absolute displacement) of the inverter reduces from $F = 3.81$ to 3.61. In this example, the design domain is discretized into 150×300 quadrilateral elements and the following parameters are implemented: $E_0 = 1$, $E_min = 10e-9$, $V^* = 0.2$, $k_{in} = 0.2$, $k_{out} = 0.005$ and $r = 10$ elements.



(a) $F = 3.81$ (b) $F = 3.77$ (c) $F = 3.61$

Figure 2: Optimized compliant mechanisms with different minimum length scale: (a) without length scale control, containing tiny hinges at structural joints; (b) with minimum length scale (equals to the size of the dashed and solid circles), considered threshold range $\eta \in (0.4, 0.6)$; (c) $\eta \in (0.3, 0.7)$.

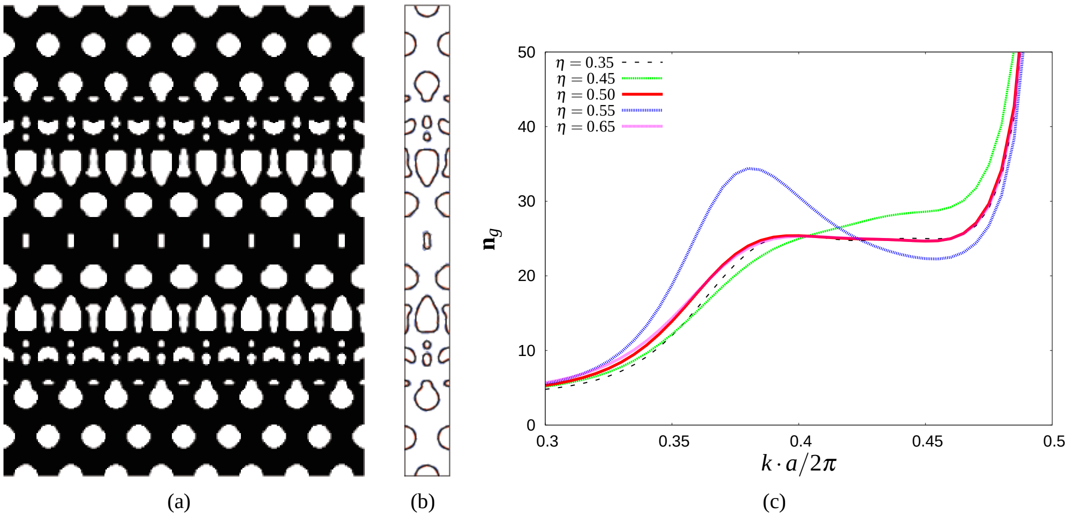


Figure 3: Optimized slow light waveguide by using both a robust formulation and geometric constraints. (a) the waveguide composed of 8 repeated cells; (b) contours in a single cell, the blueprint (in black bold lines), dilated (in blue dashed lines) and eroded design realization (in red dash-dotted lines); (c) performance of different realizations.

The second problem studies the design of a dispersion engineered slow light waveguide, which is obtained by minimizing the errors between actual group index n_g and a prescribed group index $n_g^* = 25$ in a given wavenumber range $k \in [0.3875,\ 0.4625] \cdot 2\pi/a$, where a is the width of the single cell shown in Fig. 3(b). The original problem formulation can be found in [20]. It is known that using a robust formulations cannot always guarantee the same topology for all the considered design realizations [20]. Here, by incorporating the proposed geometric constraints into the robust formulation, Fig. 3(a) shows the new optimized result containing 8 cells, in which minimum length scale in both solid and void phases are clearly identified. As shown in Fig. 3(b), the contours of the eroded, blueprint and dilated designs indicate a same topology. Equally optimized performance is achieved for the considered three designs as shown in Fig. 3(c). However, because the robust formulation here only takes three designs into account, the other intermediate realizations (e.g. $\eta = 0.45, 0.55$) still do not behave as well as the blueprint. This issue may be alleviated by including more realizations in the formulation. In this example, the design domain (one cell) is discretized into a 512×32 quad mesh. The following parameters are implemented: filter radius $r = 3.75$ elements, $\eta_d = 0.35$, $\eta_i = 0.5$ and $\eta_e = 0.65$, $\varepsilon = 10^{-6}$ and $\beta = 50$.

6. Conclusions

This paper introduces a novel approach to control minimum length scale in topology optimization results by using geometric constraints. Two numerical examples from different physical problems are presented to show the general applicability of this approach. Further investigations will be carried out on extending the current idea into maximum length scale control.

7. Acknowledgements

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8. References

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