PERGAMON

Computers and Structures 76 (2000) 347–363

Computers & Structures

www.elsevier.com/locate/compstruc

Advances in piezoelectric finite element modeling of adaptive structural elements: a survey

A. Benjeddou*

Structural Mechanics and Coupled Systems Laboratory, CNAM, 2 rue Conte, F-75003 Paris, France

Abstract

This paper makes a first attempt to survey and discuss the advances and trends in the formulations and applications of the finite element modeling of adaptive structural elements. For most contributions, the specific assumptions, in particular those of electrical type, and the characteristics of the elements are precised. The informations are illustrated in tables and figures for helpful use by the researchers as well as the designers interested in this growing field of smart materials and structures. Focus is put on the development of adaptive piezoelectric finite elements only. However, papers on other applications and active systems are also listed for completeness purpose. In total, more than 100 papers were found in the open literature. Taking this number as a measure of research activity, trends and ideas for future research are identified and outlined. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Piezoelectric finite elements; Adaptive structures modeling; Solids; Shells; Plates; Beams

1. Introduction

Since the early 70s, many finite element models have been proposed for the analysis of piezoelectric structural elements. They were mainly devoted to the design of ultrasonic transducers till the early 90s [3,4,25,49,50,63,70,107]. By the late 80s, interests have been directed towards applications in smart materials and structures [98]. During the last two decades, several review papers and bibliographies have appeared in the open literature on the finite element technology [71] and modeling of structural elements [69]. These include sandwich plates [36], thin [111] and moderately thick [34] shells, and layered anisotropic composite plates and shells [79]. However, careful analysis of these survey papers and those on the relatively new field of 'intelligent' or smart materials and structures [26,76,77] indicates that the finite element modeling of adaptive structural elements does not retain the expected attention. In fact, this highly active application area of finite element methods is in continuous growth, particularly during the last five years (Fig. 1). Hence, it gains a certain maturity so that some piezoelectric elements have become available in commercial finite element codes [67,72].

It is the objective of this paper to make a first attempt to survey and discuss the advances and trends in the formulations and applications of the finite element modeling of adaptive structural elements, namely, solids, shells, plates and beams. The underlying assumptions, in particular those of electrical type, and the characteristics of the elements such as their

* Tel.: +33-1-4027-2760; fax: +33-1-4027-2716.
E-mail address: benjedou@cnam.fr (A. Benjeddou).

PII: S0045-7949(99)00151-0

shapes, independent variables, interpolation functions degrees, and nodal degrees of freedom (dofs) are precised for the main contributions. The primary interest is the analysis of piezoelectric-based rather than other active materials-based adaptive structural finite element formulations. The informations are presented in tables and figures for helpful use by the researchers as well as the designers interested in this continuously growing field of smart materials and structures.

In the following, common theoretical formulations used for finite element development are first discussed according to the variational equations used, and the specific assumptions made to take into account the electro-mechanical coupling. Then, the piezoelectric finite element characteristics are detailed for solids, shells, plates and beams, separately. Particular attention is paid to the use or not of electric dof. Next, applications and current trends in smart finite element modeling are briefly discussed. As a closure of this survey, some ideas are outlined for future research directions. More than 100 papers are listed alphabetically at the references section. Therefore, this survey is surely incomplete and the author wishes to apologize, in advance, for any inadvertent omission of relevant publications.

2. Theoretical considerations

It is useful to recall the basic equations governing the electroelastic behavior of piezo-electric continua, which are the starting point to finite element formulations. The virtual work and energy-based formulations are then established. Next, some specific problems related to the modeling of smart structures, such as electro-mechanical coupling and induced potential representations, and common assumptions made to deal with them are discussed. Finally, conventional and advanced actuation mechanisms used in smart structures applications are outlined.

2.1. Basic piezoelectric equations

The electroelastic response of a piezoelectric body of volume Ω and regular boundary surface S, is governed by the mechanical, dynamic and electrostatic equilibrium equations,

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i \tag{1}$$

$$D_{i,i} - q = 0 \tag{2}$$

where, f_i, q, ρ, are mechanical body force components, electric body charge and mass density, respectively. σ_{ij} and D_i are the symmetric Cauchy stress tensor and electric displacement vector components. They are related to those of linear Lagrange symmetric tensor ε_{ij} and electric field vector E_i through the converse and direct linear piezoelectric constitutive equations,

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k \tag{3}$$

$$D_i = e_{ikl}\varepsilon_{kl} + \in_{ik} E_k \tag{4}$$

C_{ijkl}, e_{kij} and $\in_{ik}$ denote elastic, piezoelectric and dielectric material constants. The strain tensor and electric field vector components are linked to mechanical displacement components u_i and electric field potential φ, by the following relations,

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{5}$$

$$E_i = -\varphi_{,i} \tag{6}$$

The piezoelectric body Ω, could be subject to either essential or natural mechanical and electric boundary conditions, or a combination of them, on its boundary S,

$$u_i = U_i \tag{7a}$$

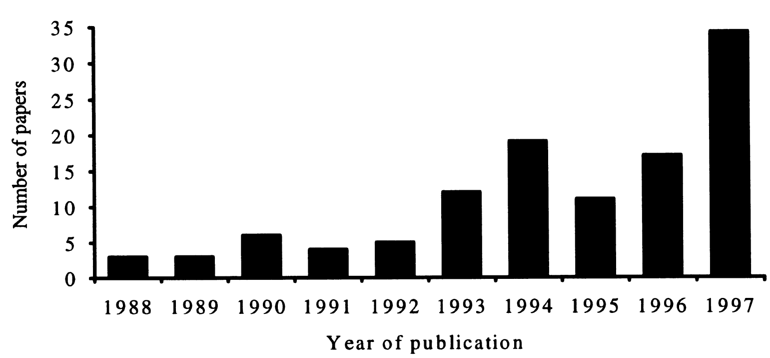


Fig. 1. Development rate of piezoelectric finite elements during the last decade.

or

$$\sigma_{ij}n_j = F_i \tag{7b}$$

$$\varphi = V \tag{8a}$$

or

$$D_i n_i = -Q \tag{8b}$$

where U_i, F_i, V, Q and n_i are specified mechanical displacement and surface force components, electric potential and surface charge, and outward unit normal vector components.

The local three-dimensional electroelastic problem consists of finding the mechanical displacement components u_i and electric potential φ satisfying Eqs. (1)–(8b) completed by adequate initial conditions.

2.2. Variational piezoelectric equations

For arbitrary space-variable and admissible virtual displacements δu_i and potential $\delta\varphi$, Eqs. (1) and (2) are equivalent to,

$$\int_\Omega \left(\sigma_{ij,\,j} + f_i - \rho\ddot{u}_i\right)\delta u_i \,\mathrm{d}\Omega + \int_\Omega \left(D_{i,\,i} - q\right)\delta\varphi \,\mathrm{d}\Omega = 0 \tag{9}$$

Integrating by parts, this equation, and using the divergence theorem, leads to

$$\begin{aligned} &-\int_\Omega \sigma_{ij}\delta u_{i,\,j}\,\mathrm{d}\Omega + \int_S \sigma_{ij}n_j\delta u_i\,\mathrm{d}S + \int_\Omega f_i\delta u_i\,\mathrm{d}\Omega \\ &\quad - \int_\Omega \rho\ddot{u}_i\delta u_i\,\mathrm{d}\Omega - \int_\Omega D_i\delta\varphi_{,i}\,\mathrm{d}\Omega + \int_S D_i n_i\delta\varphi\,\mathrm{d}S \\ &\quad - \int_\Omega q\delta\varphi\,\mathrm{d}\Omega = 0 \end{aligned} \tag{10}$$

Using the symmetry property of the stress tensor, the natural boundary conditions (7b), (8b) and the electric field-potential relation (6) give,

$$\begin{aligned} &-\int_\Omega \sigma_{ij}\delta\varepsilon_{ij}\,\mathrm{d}\Omega + \int_S F_i\delta u_i\,\mathrm{d}S + \int_\Omega f_i\delta u_i\,\mathrm{d}\Omega \\ &\quad - \int_\Omega \rho\ddot{u}_i\delta u_i\,\mathrm{d}\Omega + \int_\Omega D_i\delta E_i\,\mathrm{d}\Omega - \int_S Q\delta\varphi\,\mathrm{d}S \\ &\quad - \int_\Omega q\delta\varphi\,\mathrm{d}\Omega = 0 \end{aligned} \tag{11}$$

Substituting the constitutive equations (3) and (4) into Eq. (11) leads to the electric potential (or field)-based variational principle, which is the starting point of finite element formulations using independent variables u_i and φ. In this case, it could be seen as the sum of the conventional principle of virtual displacements (first line of (11)), and that of virtual electric potential (second line of (11)), as suggested in [30–32].

Supposing now that δu_i and $\delta\varphi$ are time-dependent and vanishing for arbitrary but fixed times t_0 and t_1, the following expression holds,

$$-\int_{t_0}^{t_1} \rho\ddot{u}_i\delta u_i\,\mathrm{d}t = \int_{t_0}^{t_1} \delta\left(\frac{1}{2}\rho\dot{u}_i\dot{u}_i\right)\mathrm{d}t \tag{12}$$

and when it is used in Eq. (11), the extended Hamilton's principle is obtained, for arbitrary space and time-variable δu_i and $\delta\varphi$ vanishing at t_0 and t_1,

$$\delta\int_{t_0}^{t_1} (T - U)\,\mathrm{d}t = 0 \tag{13}$$

T and U are the kinetic energy and extended potential energy including the electric contribution, defined by the following expressions,

$$T = \frac{1}{2}\int_\Omega \rho\dot{u}_i\dot{u}_i\,\mathrm{d}\Omega \tag{14}$$

$$\begin{aligned} U = {} & \frac{1}{2}\int_\Omega \sigma_{ij}\varepsilon_{ij}\,\mathrm{d}\Omega - \int_S F_i u_i\,\mathrm{d}S - \int_\Omega f_i u_i\,\mathrm{d}\Omega \\ & - \frac{1}{2}\int_\Omega D_i E_i\,\mathrm{d}\Omega + \int_S Q\varphi\,\mathrm{d}S + \int_\Omega q\varphi\,\mathrm{d}\Omega \end{aligned} \tag{15}$$

If the constitutive equations (3) and (4) are substituted in the first and fourth integrals of Eq. (15), Hamilton's principle (13) reduces then to the stationarity of the Lagrangian functional $L = T - U$, for arbitrary admissible δu_i and $\delta\varphi$,

$$\delta L = \delta T - \delta U = 0 \tag{16}$$

Introducing the following electromechanical energy H, and the work of external mechanical and electric body and surface forces and charges,

$$H = \frac{1}{2}\int_\Omega \left(\sigma_{ij}\varepsilon_{ij} - D_i E_i\right)\mathrm{d}\Omega \tag{17}$$

$$\begin{aligned} W = {} & \int_S F_i u_i\,\mathrm{d}S + \int_\Omega f_i u_i\,\mathrm{d}\Omega - \int_S Q\varphi\,\mathrm{d}S \\ & - \int_\Omega q\varphi\,\mathrm{d}\Omega \end{aligned} \tag{18}$$

the following relation between the extended potential energy, the electromechanical energy and work of external mechanical and electric loads is obtained,

$$U = H - W \tag{19}$$

This leads to the more common form of the variational equation (16) when relations (3) and (4) are used in

Eq. (17); i.e., for admissible δu_i and $\delta\varphi$,

$$\delta T - \delta H + \delta W = 0 \tag{20}$$

Variational equations ((11), (16) and (20)) with the constitutive relations ((3) and (4)) are the most used for piezoelectric finite element formulations. However, other variational formulations were also met in the covered literature such that proposed in [32] using the mechanical displacement u_i, electric potential φ and displacement D_i as independent variables. Therefore, the electric field is computed from the direct constitutive equation (4) but, in terms of strains and electric displacements rather than from the electric field-potential relation (6). The latter was introduced in (10) as a constraint via a Lagrange multiplier. The Hu–Washizu principle was also modified in [87,88] to include virtual work done by electric forces.

2.3. Specific problems and common assumptions

This sub-section focuses on the representation of the electromechanical coupling inherent to piezoelectric materials, the widely used assumption of linear potential and its effects on the piezoelectric coupling representation, parallel polarization and applied electric field assumption, and the various advanced actuation mechanisms used in smart structures applications, in particular, the recent shear one.

2.3.1. Electromechanical coupling

The major feature added by the piezoelectric material to the standard structural finite element modeling is its electromechanical coupling. Moreover, due to the fact that the electric charge is distributed on both top and bottom surfaces of a piezoelectric patch, considering the electric contribution in the discretization procedure is a hard task, particularly for two and one-dimensional mid-plane conventional formulations.

The full electromechanical coupling and surface characteristics could be handled through three-dimensional finite element formulations with an extended nodal dofs vector containing both mechanical and electric dofs. However, those done assuming through-thickness linear variation of the electric potential would neglect the induced potential and the electromechanical coupling will be partial, as will be shown later. It is thought that a quadratic through-thickness variation of the electric potential would enhance the electromechanical coupling. In fact, it was shown that the asymptotic electric potential of a short circuited thin piezoelectric plate is quadratic in thickness [75]. This was confirmed for shear flexible plates by higher order 2D-theories [16,20], and was assumed for brick [58,108] and plate [16] finite element formulations.

The fully coupled electromechanical linear system describing the behavior of a smart structure with electric nodal or element dofs representation is often uncoupled through their Guyan condensation [3,16,21–23,83,84,99,101,102,108]. This leads to an increase of the structure's stiffness and an additional electric load vector. It is thought that these results are equivalent to the modification of the constitutive equations and the additional electric loads obtained without the use of electric dofs, but considering the induced potential [11,12,74]. The main difference is that, the first condensation is made on the discretized equations, whereas the second is on the variational formulation.

The coupled electromechanical system can also be taken into account by iterative solution between direct and converse effect equations [30]. This method has the advantage to avoid the use of enlarged nodal unknown vector, but still neglects the induced potential.

2.3.2. Induced electric potential

A widely used assumption is the through-thickness linear variation of the electric potential. Consequently, the induced potential is systematically neglected. To illustrate the influence of this hypothesis on the piezoelectric coupling, the electric potential is decomposed into a linear part φ_0, known from the prescribed potentials, and an unknown part ϕ, representing the induced potential [74],

$$\varphi = \varphi_0 + \phi \tag{21}$$

Using this decomposition, together with the constitutive equations (3) and (4), and after some manipulations, Eq. (11) becomes,

$$\begin{aligned}
&\int_\Omega \left[C_{ijkl}\varepsilon_{kl}\delta\varepsilon_{ij} - e_{ikl}\varepsilon_{kl}\delta E_i\right] \mathrm{d}\Omega - \int_\Omega \left[e_{kij}E_k(\phi)\delta\varepsilon_{ij} + \right.\\
&\quad \left. \in_{ik} E_k(\phi)\delta E_i\right] \mathrm{d}\Omega + \int_\Omega \rho\ddot{u}_i\delta u_i \,\mathrm{d}\Omega \\
&\quad = \int_\Omega f_i\delta u_i \,\mathrm{d}\Omega + \int_S F_i\delta u_i \,\mathrm{d}S + \int_\Omega q\delta\varphi \,\mathrm{d}\Omega + \int_S Q\delta\varphi \,\mathrm{d}S \\
&\quad + \int_\Omega \left[e_{kij}E_k(\varphi_0)\delta\varepsilon_{ij} + \in_{ik} E_k(\varphi_0)\delta E_i\right] \mathrm{d}\Omega
\end{aligned} \tag{22}$$

It is clear then, that the second integral of the left hand side (l.h.s.) of (22) vanishes when the induced potential ϕ (or field $E_k(\phi)$) is neglected. The piezoelectric effect is then represented only by a partial electromechanical coupling (second term in the first integral of the l.h.s.) and an equivalent electric load vector (last integral on the r.h.s.). Moreover, since for actuation problems only, the variations of the electric field components are zero and electric charges are often not considered, the above equation reduces to,

$$\int_{\Omega} C_{ijkl}\varepsilon_{kl}\delta\varepsilon_{ij}\,\mathrm{d}\Omega + \int_{\Omega} \rho\ddot{u}_i\delta u_i\,\mathrm{d}\Omega = \int_{\Omega} f_i\delta u_i\,\mathrm{d}\Omega + \int_{S} F_i\delta u_i\,\mathrm{d}S + \int_{\Omega} e_{kij}E_k(\varphi_0)\delta\varepsilon_{ij}\,\mathrm{d}\Omega \tag{23}$$

Hence, the piezoelectric effect is represented by the equivalent electric load vector (last term in (23)) and an increase of the structure's stiffness and mass, sometimes neglected [40]. This is often used for the actuator variational formulation. Above equation can also be obtained by considering the converse constitutive equation (3), with a linear potential assumption, in a conventional mechanical variational principle. An equation similar to (23) could also be obtained by the so-called thermal analogy approach. It is based on the resemblance between thermoelastic and converse piezoelectric constitutive equations when Eq. (3) is written in the form,

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \Lambda_{kl}); \quad \Lambda_{kl} = C^{-1}_{ijkl}e_{mij}E_m = d_{mkl}E_m \tag{24}$$

Λ_{kl} is interpreted as initial (or induced actuation) strain tensor components, and are often computed as thermal strains using this analogy. Hence, thermoelastic finite element analysis codes could be used to investigate smart structures.

2.3.3. Piezoelectric actuation mechanisms used in smart structures applications

Most piezoelectric finite element formulations assume electric field and poling direction along the piezoelectric patch thickness. Only longitudinal strains or stresses could then be induced by monolithic piezoelectric materials. This is their conventional extension actuation mechanism, which could be seen as the basis of the so-called pin-force or engineering approach, often used at the early 80s for the validation of the piezoelectric converse effect. However, it could be shown that for perpendicular electric field and polarization, shear resistant monolithic piezoelectric materials could induce transverse shear strains or stresses [11,12]. Comparison of both the mechanisms on cantilever smart beams [13,14] indicates that only thin extension actuators are efficient. On the contrary, shear actuators are more efficient for a medium thickness range. Moreover, for stiffer basic structure, shear actuation mechanism presents better performance than extension one. It was also found that shear actuated beam is less deformed. Hence, the bending stress is also smaller. This is an advantage for brittle piezoceramics. These performances were observed numerically only. Therefore, they need to be confirmed experimentally for different structural elements. Besides, current commercially available piezoceramics are not optimized for their shear piezoelectric response. They were optimized for their extension piezoelectric properties only. This relatively new concept of shear actuation merits more investigations and is thought to be promising.

Another common assumption is that only transverse components of the electric field and displacement are retained for most piezoelectric finite element formulations. This implicitly supposes that the in-plane components are much smaller and could be neglected. However, using interdigitated electrodes, transverse actuation could also be introduced [31,32]. A complex poling pattern results in the actuator due to induced in-plane components of electric field which should be accounted for any model.

By sandwiching a piezoelectric layer between off-axis laminae, such as in the piezoelectric fiber composites [1,31,32], twisting deformation can be induced through transformed piezoelectric constants. This twisting is caused by the extension-twisting coupling. Four sectored sensors/actuators can also produce torsional twisting beside extensional and bending actuations, leading to a multi-axial active control system [91].

3. Finite element development

During the last decade, finite element modeling of smart structures has attracted numerous researchers and has become a major area of research (Fig. 1). Early investigations were devoted to 3D elements with nodal electric potential dofs. They take account of the surface characteristics and full electro-mechanical coupling, inherent to piezoelectric patches. However, it was found that these were too thick to modelize very thin structures. Hence, attention was directed to 2D elements, despite the difficulty of the conventional midplane formulations to take into account potentials on upper and lower surfaces. This motivates the recent development of one- and two-dimensional elements free of electric dofs (Fig. 2). Standard finite elements are then used to compute mechanical behavior (displacement, strains, stresses), and electrical quantities (charge, current, potential) are deduced from the

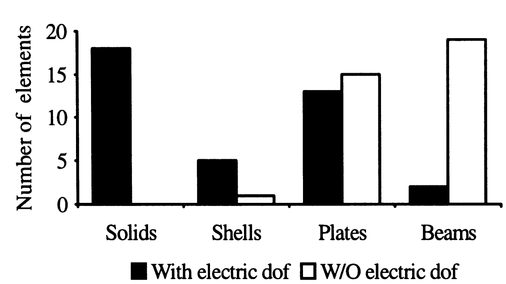
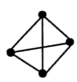
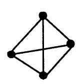
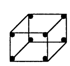
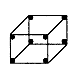
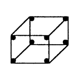
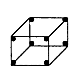
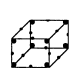
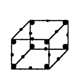


Fig. 2. Trends in development of piezoelectric finite elements.

specific sensing/actuation relations. This is often achieved through several simplifications (cf. discussion in above section).

In the following, finite element characteristics such as their shapes, variables, nodal/element dofs are detailed separately for solid, shell, plate and beam elements. Fig. 2 indicates that nearly all solid and shell finite elements have electric dofs. However, for plates and beams, electric dofs are often avoided.

Table 1
Characteristics of some piezoelectric solid finite elements

Shape	Authors (year)	Approximations	Nodal dofs	Total dofs	Comments
 Tet4	Allik, Hughes (1970)	u,v,w : linear φ : linear	u, v, w φ	16	-Static condensation of the electric dofs -Application to acoustic transducers
 Tet4	Ghandi, Hagood (1997)	u,v,w : linear φ : linear	u, v, w φ	16 + Intern. dofs	- Nonlinear constitutive equations - Electric displacement-based formulation - Element internal dofs to represent phase transition and remnant polarization
 Hex8	Tzou, Tseng (1990)	u,v,w : linear +quadratic incompatible modes,φ : linear	u,v,w + internal dofs φ	32	- Static condensation of the incompatible modes at the element level and electric dofs after elements assembly -Application to intelligent plates
 Hex8	Ha, Keilers, Chang (1992)	u,v,w : linear φ :linear+ quad. incom. modes	u, v, w φ + int. dofs	32	- Equivalent single layer model - Static condensation of the incompatible modes at the element level
 Hex8	Ghandi, Hagood (1996)	u,v,w : linear φ : linear	u, v, w φ	32 + intern. dofs	- Nonlinear constitutive equations - Iterative F. E. solution - Element internal dofs to represent phase/polarization state of the material
 Hex8	Chin, Varadan, Varadan (1994)	u,v,w : linear φ : linear	u, v, w φ	32	-Lagrange's method for matrix formulation - Application to periodic piezoelectric arrays subjected to fluid-loading
 Hex20	Allik, Webman (1974)	u,v,w: quadratic φ : quadratic	u, v, w φ	80	- Application to response of sonar transducers
 Hex20	Koko, *et al.* (1997)	u,v,w: quadratic φ : quadratic θ : quadratic	u, v, w φ θ	100	- Thermo-piezo-electric constitutive laws - Guyan condensation of the electric dofs - Application to beams and plates

3.1. Solid elements

Three-dimensional piezoelectric solid elements generalize those used in structural mechanics through an extended dofs vector, i.e., containing additional electric dofs. They were either tetrahedral [3,32] or brick elements [5,24,25,31,38,55,56,58,65,66,90,99,100, 102,105,108]. The electric potential was supposed linear except for the 20-node hexahedron for which it is quadratic [4,58,108]. Thermal piezoelectric effects [76] were considered for the eight-node [105], 18-node [5] and 20-node [58] hexahedral elements. Nonlinear constitutive relations were considered for the four-node tetrahedral [32] and eight-node hexahedral [31] element. This was retained for more accurate representation of the piezoelectric material response at high electric fields. These elements have additional internal variables to represent the phase/polarization state of each element. They are adapted at each simulation step, based on a phenomenological model. Beside, internal dofs were added to enhance the behavior of the eight-node brick element when used for very thin structures. Hence, quadratic incompatible modes were included for mechanical displacements in [99–101,104] and for mechanical displacements and electric potential in [31].

Early elements did not deal with layered structures [3,24,25,90,99,100,108]. These were then handled through the equivalent single layer model [38]. The electric dofs are also often condensed by the Guyan procedure to uncouple the electro-mechanical problem [3,38,58,99–101,105]. Detailed description of some piezoelectric solid elements is given in Table 1. It

Table 2
Characteristics of some piezoelectric shell finite elements

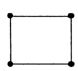
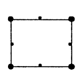
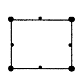
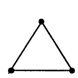
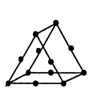
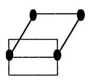
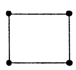
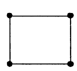
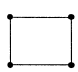
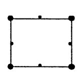
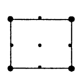
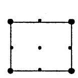
Shape	Authors (year)	Approximations	Nodal dofs	Total dofs	Comments
 Shell4	Lammering (1991)	u,v,w : linear β_x,β_y : linear φ : linear	u,v,w β_x,β_y φ_u, φ_l	28	-Shallow shell shear (+ RI) theory -Equivalent single layer model for 3-layers -Virtual displacemnt & potential principle -Upper/lower nodal electric potential dof
 Shell8	Thirupathi, Seshu, Naganathan (1997)	u,v,w: quadratic β_x,β_y: quadratic	u,v,w β_x,β_y	40	- 3D-degenerated shell theory - Piezo. effect as initial strain problem - Static analysis of smart turbine blades
 Shell-axi8	Guo, Cawley, Hitchings (1992)	u,v,w : quadratic φ : quadratic	u, v, w φ	32	- Generalized stress, strain, displacement, force variables (mechanical + electric) - No electric dofs condensation - Modal analysis and harmonic response
 Shell-axi3	Varadan, Chin, Varadan (1993)	u,w: linear φ : linear	u, w φ	9	- Potential energy including both elastic and electric energies - Lagrange formulation of motion eqns. - Application to Mooney transducer
 Shell12	Tzou, Ye (1994)	u,v,w φ in-plane:quadratic thickness:linear	u,v,w φ	48	- Layer-wise constant shear angle theory - Degenerated 3D-shell theory - Laminated piezoelectric shell continuum - Energy functions for the displacement and electric potential functional

Table 3
Characteristics of some piezoelectric plate finite elements with electric dofs

Shape	Authors (year)	Approximations	Nodal dofs	Total dofs	Comments
 2D/3D	Heyliger et al. (1994)	u,v: piecewise linear w variable: constant/linear φ : linear	u^{ji}, v^{ji}, w^{ji} φ^{ji} j=1,..,N i=1,..M	variab-le	- Discrete layer theories with RI : a) through thickness: 1D polynomials b) in-plane: 2D functions -Static condensation of electric dofs - Induced potential , Thermal environment
 Quad4	Suleman, Venkayya (1995)	u,v,w: bilinear $\theta_x,\theta_y,\theta_z$:bilinear φ : linear	u,v,w $\theta_x,\theta_y,\theta_z$ $\varphi_1,..,\varphi_{np}$	24 + np	- Mindlin plate element (C^0) + uniformly reduced numerical integration and hourglass stabilization - 1 potential dof per piezoelectric layer
 Quad4	Veley, Rao (1995)	u,v,w: bilinear φ : linear	u,v,w φ	16	- Combination of a 3D piezoelectric element (Hex element defined by pseudo-nodes for top/bottom surfaces) and an ACLD element (Quad/Hex/Quad) - Application to ACLD treatments
 Quad4	Chen, Wang, Liu (1997)	w: cubic φ : linear	w, $w_{,x}$, $w_{,y}$ φ	16	- Kirchhoff plate element (bending only) - Guyan reduction for the electrical dofs
 Quad8	Ray, Bhattacharyya, Samanta (1994)	u,v,w, l_x,l_y,l_z, m_x,m_y,m_z, $n_x,n_y,\varphi^1,\varphi^n$: quadratic	all	104	- 11 variables higher-order shear deformable theory - Linear potential in thickness - Nodal electric dofs condensed
 Quad9	Carrera (1997)	u,v,w: quadratic $\sigma_{\alpha 3}$: quadratic φ : quadratic	u,v,w,β_α D_α, $_{\alpha=1,2}$ $R_{\alpha 3}$ $\sigma^t_{\alpha 3}$,$\sigma^b_{\alpha 3}$	variab-le + φ^t, φ^c, φ^b	-Multilayered plates including piezo-layers -Higher order shear deformation theory (but σ_{33}=ε_{33}=0, and standard C^0 F.E.) - In-plane displacement: zig-zag effect - Interlaminar equilibrium and top/bottom transverse shear stress conditions fulfilled - Stress dofs eliminated at the layer level - Layer voltage dofs condensed/layer level
 Quad9	Yin, Shen (1997)	u,v,w β_x,β_y ψ=-E_3 quadratic	u,v,w β_x,β_y ψ=-E_3	54	-Mindlin plate theory (C^0) - Linear voltage but transverse field dof - Damage detection (strain sensing)

appears that quadratic tetrahedral element was not proposed. It is also thought that quadratic elements would be expected to behave better than linear ones, since the induced potential is taken into account.

3.2. Shell elements

Only few piezoelectric shell elements were found in the literature (Table 2). A four-node shell element

Table 4
Characteristics of some piezoelectric plate finite elements without electric dofs

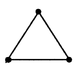
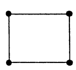
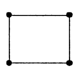
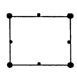
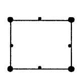
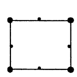
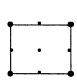
Shape	Authors (year)	Approximations	Nodal dofs	Total dofs	Comments
 Tria3	Shen, Sharpe Jr. (1997)	u,v :linear w: Discrete Kirchoff Theory (DKT)	u,v w, β_x,β_y	15	- Aeroelastic analysis of hypersonic panel flutter supression (Non linear) -Thermal effect considered - Piezoelectric effect by thermal analogy
 Quad4	Hwang, Hwang, Park (1994)	w: DKT approximation	w, β_x,β_y	12	- Discrete Kirchhoff bending theory - Equivalent single layer model - Modal reduction technique - Piezoelectric effect by thermal analogy
 Quad4	Baz, Ro (1996)	u_i, v_i : bilinear i=1,3,5 w : bicubic	u_i,v_i i=1,3,5 w $w_{,x}$,$w_{,y}$	36	- 3-layer theory; 1,5 : constraining layers, 3: base plate, sensors+plate= 1 layer - 7 layers reduced to 5 layers for ACLD - Shear strain neglected in the piezos.
 Quad8	Battisti, Rotunno, Sermoneta (1997)	w: quadratic (incomplete) β_x,β_x: quadratic (incomplete)	w β_x,β_x	24	- Reissner-Mindlin plate element (C^0) - Equivalent single layer model - Modal synthesis - No shear effect in the piezo-layers - Electrostatic energy neglected
 Quad8	Liu, *et al.* (1997)	u,v,w: quadratic (incomplete) β_x,β_x: quadratic (incomplete)	u,v,w β_x,β_x	40	- Reissner-Mindlin plate element (C^0)+RI - Equivalent single layer model - Modal analysis technique - Electrostatic energy neglected
 Quad8	De Sciuva, Icardi, Villani (1997)	u,v: cubic in each layer w : cubic (but constant through thickness)	u,v w, $w_{,x}$,$w_{,y}$ g_1, g_2 shear rotations	56	- Geometrically nonlinear multilayer theory with shear effect and off-axis angle - Inter-laminar TS stresses continuity and top/bottom stress-free conditions satisfied - Principle of virtual displacement + reduced constitutive equations (σ_{33}=0)
 Quad9	Chandrashekha-ra, Tenneti (1995)	u,v,w: quadratic β_x,β_x: quadratic	u,v,w β_x,β_x	45	- First-order shear deformation theory (C^0) - Equivalent single layer model - Piezo. mass, stiffness, thermal expansion - Electrostatic energy neglected

Table 5
Characteristics of some piezoelectric beam finite elements with electric dofs

Shape	Authors (year)	Approximations	Nodal dofs	Total dofs	Comments
	Shen	u : linear	u	8	- Timoshenko beam theory using an
Beam2	(1994)	w : cubic	w, $w_{,x}$		extended Hu-Washizu Principle (Mixed)
with		Hermite			- Special assembly actuator/beam/sensor
offset		β : linear	β	+	elts. for longitudinal-bending coupling
nodes				V_u,V_b	-Local Rayleigh-Ritz method for piecewise
					continuous fit to the deflection shape
					- Upper/lower potential dofs per element
	Carpenter	u : linear	u	8	- Euler-Bernoulli strain model
Beam2	(1997)	v cubic Hermite	v, $v_{,x}$		- Includes electric field work
		φ linear	φ		- Axial vibration and plane bending

extending the shallow shell shear deformation theory was proposed using an equivalent single layer model for a three-layer shell [60,61]. Upper and lower nodal electric potential dofs were chosen to represent surface characteristics of the piezoelectric layer. Reduced integration (RI) was used to avoid shear locking. An eight-node quadrilateral shell element [95], free of electric dofs, was also formulated using the 3D-degenerated shell theory. The piezoelectric effect was treated as an intial strain problem.

An axisymmetric three-node triangular shell element was developed to study Mooney transducers [107]. An eight-node quadrilateral shell element was also proposed to predict vibration characteristics of piezoelectric discs [35]. These elements do not deal with multi-layers. Therefore, a 12-node 3D-degenerated shell element, with layer-wise constant shear angle, was formulated in [105]. Displacements and electric potential were supposed in-plane quadratic and through-thickness linear.

Plate elements could also be adapted to modelize shell structures but after a geometric transformation, to take into account the shell curvatures. 3D solid elements were also used in the literature for adaptive shell modeling. However, it is thought that more research efforts are needed to better understand the influence of the curvatures on the piezoelectric actuators and sensors. Investigations could also be directed to shear actuated shells, in particular, in presence of internal fluid (structural acoustics).

3.3. Plate elements

As discussed above, the representation of the surface electric characteristics of the piezoelectric layers is somewhat difficult for conventional mid-plane two-dimensional formulations. This explains the dominance of electric dofs-free techniques for recent piezoelectric finite elements development. However, for plates, Fig. 2 indicates that both techniques (with and without electric dofs) were used, contrary to the solid and shell elements. Hereafter, these are discussed separately.

3.3.1. With electric dofs representation

Only quadrilateral elements were proposed for finite element modeling of adaptive plates (Table 3). A four-node element, with in-plane variable dofs, was formulated using discrete layer theories for laminated piezoelectric plates [4,20,62,84]. The potential and in-plane displacements were piecewise linear whereas, the deflection could be either constant or linear. This formulation has the advantage to represent a quadratic potential, thanks to the numerical through-thickness finite element subdivisions.

A four-node element was proposed for active control of Kirchhoff–Love plate bending vibrations [21–23]. Another one was obtained by combination of a 3D piezoelectric element, defined by pseudo-nodes for top and bottom surfaces, and an ACLD element [109,110]. Also, an eight-node quadrilateral element was formulated using a higher order shear deformable displacement theory, but assuming through-thickness linear variation of the electric potential [78]. Previous el-

Table 6
Characteristics of some piezoelectric beam finite elements without electric dofs

Shape	Authors (year)	Approximations	Nodal dofs	Total dofs	Comments
	Robbins, Reddy	CBT, u : linear	u	6	- 2 equivalent single layer (CBT/SDBT) &
	(1991)	w : cubic	w, $-w_{,x}$		2 multi-layer models (MLBT1,2)
Beam2	SDBT:	u,w,β : linear	u,w,β	6	- Piezo. by thermal analogy: $\sigma_1=E_1(\varepsilon_1-\Lambda_1)$
	MLBT1:	u: p/w linear	$u_{j,j=1,N+1}$	2N+2	- Principle of virtual displacement without
		w : constant	w		electrical contribution
	MLTB2:	u,w: p/w linear	u_j, w_j	4N+4	- N= finite element subdivisions
			$j=1,..,N+1$		(>= number of material layers)
	Baz, Ro	u_1,u_3, linear	u_1,u_3	8	- PVDF/ ViscoElastic/ PVDF/Beam struct.
	(1994)	w :cubic	w, $w_{,x}$		- 3 layer theory (1:actuator, 3:beam+piezo)
Beam2		Hermite			- No shear in PVDF facings and the beam
					- Damping:Augmented Thermodyn. Fields
	Benjeddou,	w : cubic	w, $w_{,x}$	8	- Three-layer theory: Euler-Bernoulli
Beam2	Trindade,	Hermite	&		facings (f) and Timoshenko core(c)
	Ohayon	$\bar{u}_c$, $\tilde{u}_c$	$\bar{u}_c$, $\tilde{u}_c$		- Variables=mean and relative axial
	(1997, 1999)	or	or		displacements and deflection
		$\bar{u}_f$, $\tilde{u}_f$: linear	$\bar{u}_f$, $\tilde{u}_f$		- Induced potential in piezo. facings
					- Shear actuation mechanism: piezo. core
	Islam, Craig	u: linear	u	6	- Euler-Bernoulli beam theory (C^1)
	(1994)	w: cubic	w, $-w_{,x}$		- Equivalent single layer model
Beam2		Hermite			- Pin force/moment engineering approach
					- Application to damage detection
	Smyser,	u: linear	u	8	- Shear deformation theory
	Chandrashekara	w:cubic	w, $-w_{,x}$		- Equivalent single layer model
Beam2	(1997)	Hermite	β		- Modal superposition/Neural network
		β: linear			- No electric contribution
	Surace,	u: linear	u,w, $w_{,x}$	10	- Timoshenko beam: $w=w_b+w_s$
	Cardascia,	w, $w_{b,x}$: cubic	$w_{b,x}$,		- Principle of virtual displacement +
Beam2	Anghal (1997)	Hermite	$w_{b,xx}$		converse constitutive equations
	Aldraihem,	T: w cubic	w		- Timoshenko vs. Euler-Bernoulli bending
Beam2	Wetherhold,	β quadratic	β	4	- No bending-twisting coupling
	Singh (1997)	E-B: w cubic	w, $-w_{,x}$		- Piezo by Engineering approach
	Lesieutre, Lee	u, β (β_A)	u, β	13	- No-shear in piezo/beam
	(1996)	quadratic	(β_A)		- Shear angle:$\beta=\beta^E+\beta^A$, A=Anelastic
Beam3		w: cubic	w, $w_{,x}$		- Additional Anelastic Displ. Field dof
		Hermite			- Piezo.: by blocked (strain=0) stress work.
	Aldraihem,	w : quadratic	w	9	- bending-twisting Timoshenko laminate
	Wetherhold	θ,β,α: linear	θ,β,α		- Equivalent single layer model
Beam3	(1997)	β,α: rotations/y			- St. Venant and warping torsion permitted
		θ:twisting angle			- PZT/Ep composite (bending actuation)

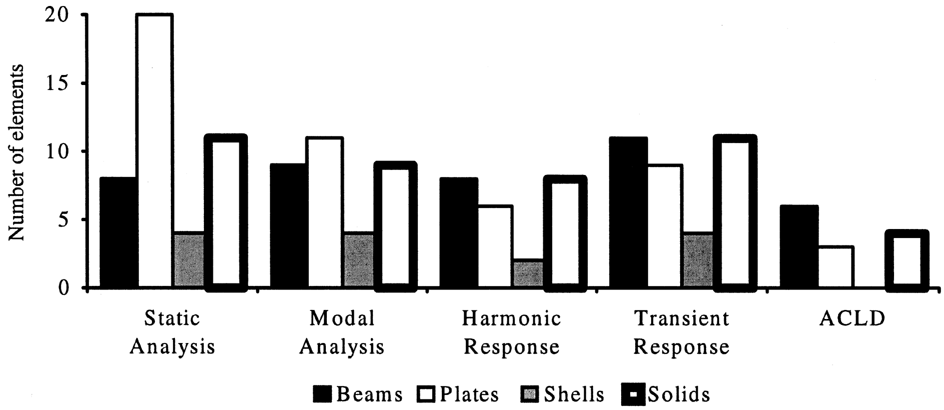


Fig. 3. Current trends in smart finite element development.

ements have electric potential nodal dofs. Nevertheless, a Mindlin plate element with one potential dof per piezo-electric layer, and using uniform reduced numerical integration and hourglass stabilization was also proposed in Ref. [93].

A quadratic nine-node element, formulated on the basis of a higher order shear deformation theory, was suggested in Ref. [16]. It fulfills zig-zag effect for in-plane displacements, interlaminar equilibrium and top/bottom transverse shear stress conditions. Beside, the electric potential was assumed quadratic in the plate thickness and is described at the layer level by top, central and bottom electric potential dofs. Stress and electric dofs were condensed at the layers system level. The only piezoelectric finite element with the transverse electric field as electric dof, was proposed in Ref. [112] using a Mindlin plate theory and through-thickness linear voltage.

It is worthy to note that three-layer theories were not applied to piezoelectric plate finite elements with electric dofs. Also, shear actuation mechanism was not studied for smart plates. This may be due to the difficulty to handle in-plane polarized patches to be used for this mechanism.

3.3.2. Without electric dofs representation

Most electric dofs-free elements used an equivalent single layer model for multilayer piezoelectric plates [6,17–19,39,45–47,51,52,68,85,86] (Table 4). Three-layer theory was retained to modelize a seven-layer ACLD plate system [10]. In fact, the piezosensors and plate were assumed to consitute a shear-free single layer. Geometrical non linearity [29] and thermal expansion [19,86] of the piezoelectric patches were also considered.

In above elements, the piezoelectric effect is present via its equivalent electric load, given by the thermal analogy approach [18,29,45–47,89] or the converse constitutive relation [6,19,39,59,68]. Classical discretized linear systems with additional force term are get for the actuator equation. For the sensor equation, the direct constitutive relation is integrated over electroded surfaces to obtain sensed electric charge, then electric current and potential are deduced through control gains. A control law is then used to close the loop.

It is thought that more investigations are needed to evaluate the shear effect on the piezoelectric actuation/sensing performance. Sandwich plate formulations with in-plane polarized piezoelectric cores would be very promising, as was shown for beams [11,12].

3.4. Beam elements

There are few piezoelectric beam elements with electric dofs representation (Table 5). It may be due to the fact that the electric dofs could be easily condensed on the continuum formulation level for one-dimensional space equations.

The Hu-Washizu variational principle was extended to include electric loads [87,88] in order to formulate a Timoshenko beam element with offset nodes. Beside the mechanical dofs, upper and lower electric potential dofs per piezoelectric element were considered. An Euler–Bernoulli element with electric potential nodal dofs was also developed for axial vibration and bending control [15].

Electric dofs-free beam finite elements are numerous (Table 6). Four layer-wise elements based on thermal analogy approach were formulated early in the 90s [82]. They are two equivalent single-layer models, representing classical and shear beam theories, and two multi-layer models with in-plane piecewise linear axial displacement and constant or cubic Hermite deflection. A number of through-thickness finite element subdivisions, greater or equal to material layers, could be considered. As indicated for the corresponding plate

Table 7
Piezoelectric finite elements found in the covered literature

Elements	Shape and approximations	With electric dofs	Without electric dofs
Solid	Four-nodes linear tetrahedron	Available	Not available
	Eight-nodes linear hexahedron	‖	‖
	20-nodes quadratic hexahedron	‖	‖
Shell	Three-nodes linear axisymmetric flat triangle	‖	‖
	Eight-nodes quadratic axisymmetric quadrangle	‖	‖
	Four-nodes linear flat quadrangle	‖	‖
	Eight-nodes 3D-degenerated quadratic quad.	Not available	Available
	12-nodes 3D-degenerated quadratic prism	Available	Not available
Plate	Three-nodes linear triangle	Not available	Available
	Four-nodes linear quadrangle	Available	‖
	Eight-nodes quadratic quadrangle	‖	‖
	Nine-nodes quadratic quadrangle	‖	‖
Beam	Two-nodes linear element	‖	‖
	Three-nodes quadratic element	Not available	‖

element [41,84], it has the advantage to better represent the induced potential and the transverse shear behavior.

Sandwich theory was used to formulate a two-node element of ACLD beam systems [8,9,81,106]. The shear effect was neglected for the piezo and basic beam. The axial displacements of the latter layers and their deflection were retained as mechanical dofs. However, in Refs. [11,12], the shear effect was considered for piezoelectric cores of sandwich piezoelectric beams. The mean and relative axial displacements of the core [11] or the faces [12] and their deflection are retained as mechanical dofs. Comparisons of both models [14] for the extension and shear actuation mechanisms [13] were performed. It was found that the former induces boundary point actuation loads, whereas the latter gives distributed actuation loads, and has better performance for stiffer base structure and for medium thickness range. These results were also confirmed for active control [97] and ACLD treatments [96].

Timoshenko and Bernoulli–Euler beam finite elements were proposed in [2,27,48,73,90,94]. They were extended to include Saint-Venant and warping torsions in PZT/Ep composites for bending vibrations control [1]. The deflection was assumed quadratic and the rotational and twisting angles are also dofs. A quadratic variation of the axial displacement and shear angle in conjunction with a cubic Hermite deflection were also assumed for a three-node beam element [64]. It was based on thermal analogy and a three-layer theory with shear-free piezo/beam layers. An elastic part of the shear angle was used as additional dof to represent a time-domain viscoelastic model for ACLD treatments.

4. Applications and current trends

Careful open literature analysis indicates that finite element modeling applications were mostly devoted to static, modal, harmonic and transient linear behavior of adaptive plates and beams (Fig. 3). An active area of research during the last five years was also the active constrained layer damping control. It consists of adding to or replacing the conventional elastic constraining layer of the passive sandwich damping treatment by an active layer. The sensor could be either an additional piezoelectric layer or a strain gauge. This relatively new concept combines the advantages of both passive and active treatments in a unique system, in particular, safety and stability of the control device.

Recent finite element analyzes were directed to thermal effects [19,58,62,104], active noise control [7,54,57,80,90,91], damage detection due to composite delamination [48] or low velocity impact [112], active buckling control [18], geometric or/and material non linearities [31,32,98,103], anisotropy and non homogeneity [33], active fiber composites [1,32,90,91] and flutter supression [89].

It is worthwhile to notice that finite element techniques were also applied with other active materials such as in electrostrictive [43,44] and magnetostrictive [53] systems. The boundary finite element technique was also proposed for piezoelectric solids [42,75]. Beside, several researchers have also simply used general purpose finite element commercial codes to check analytical or experimental analyses. These were beyond the scope of this overview and have not been cited here.

5. Conclusions

Advances in finite element modeling of smart structural elements, during the last decade, have been presented. It was found that, although a relative maturity has been reached, some topics have not received much attention. In particular, there is a lack of 2D curved and ACLD shell finite elements, and some quadratic elements with electric dofs representation (Table 7). Also, shear actuation mechanism, present in perpendicular polarization and applied electric field conditions, was not investigated for other structural elements than beams. In contrary to external fluid-loaded structures, internal fluid-loaded ones were not sufficiently investigated. These are some themes, beside those outlined in the previous sections, to which future developments would be directed.

References[1]

[1] Aldraihem OJ, Wetherhold RC. Mechanics and control of coupled bending and twisting vibration of laminated beams. Smart Mater Struct 1997;6:123–33.
[2] Aldraihem OJ, Wetherhold RC, Singh T. Distributed control of laminated beams: Timoshenko theory vs. Euler bernoulli theory. J Intell Mater Syst Struct 1997;8:149–57.
[3] Allik H, Hughes TJR. Finite element method for piezoelectric vibration. Int J Num Methods Engrg 1970;2:151–7.
[4] Allik H, Webman KM, Hunt JT. Vibrational response of sonar transducers using piezoelectric finite elements. J Acoust Soc Am 1974;56:1782–91.
[5] Bahrami H, Tzou HS. Design and analysis of a precision multi-dof placement device. In: Brei D, Sirkis J, editors. Adaptive Struct Mater Syst ASME 1997;AD-54:1–7.
[6] Battisti CO, Rotunno M, Sermoneta AA. finite element model for electrodynamic analysis of sandwich panel. In: Santini P, Rogers CA, Murotsu Y, editors. Seventh Int. Conf. Adaptive Struct. Basel: Technomic, 1996. pp. 449–62.
[7] Baz A. Active noise control of piston-cavity systems. In: Brei D, Sirkis J, editors. Adaptive Struct Mater Syst ASME 1997;AD-54:121–131.
[8] Baz A, Ro J. Performance characteristics of active constrained layer damping. Schock Vibr 1994;2:33–42.
[9] Baz A, Ro J. Optimum design and control of active constrained layer damping. J Vibr Acoust 1995;117:135–44.
[10] Baz A, Ro J. Vibration control of plates with active constrained layer damping. Smart Mater Struct 1996;5:272–80.
[11] Benjeddou A, Trindade MA, Ohayon RA. A unified beam finite element model for extension and shear piezoelectric actuation mechanisms. J Intell Mater Syst Struct 1997;8:1012–25.
[12] Benjeddou A, Trindade MA, Ohayon R. New shear actuated smart structure beam finite element. AIAA J 1999;37(3)1378–83.
[13] Benjeddou A, Trindade MA, Ohayon R. Piezoelectric actuation mechanisms for intelligent sandwich structures. Smart Mater Struct, in review.
[14] Benjeddou A, Trindade MA, Ohayon R. Two models for two actuation mechanisms: a comparison. In: Pamplona D et al., editors. Appl. Mech. in the Americas. Rio de Janeiro: AAM and ABCM, 1999;8:1545–1548.
[15] Carpenter M. Using energy methods to derive beam finite elements incorporating piezoelectric materials. J Intell Mater Syst Struct 1997;8:26–40.
[16] Carrera E. An improved Reissner–Mindlin type model for the electro-mechanical analysis of multilayered plates including piezo-layers. J Intell Mater Syst Struct 1997;8:232–48.
[17] Chandrashekhara K, Agarwal AN. Active vibration control of laminated composite plates with piezoelectric devices: a finite element approach. J Intell Mater Syst Struct 1993;4:496–508.
[18] Chandrashekhara K, Bhatia K. Active buckling control of smart composite plates-finite element analysis. Smart Mater Struct 1993;2:31–9.
[19] Chandrashekhara K, Tenetti R. Thermally induced vibration suppression of laminated plates with piezoelectric sensors and actuators. Smart Mater Struct 1995;4:281–90.
[20] Chattopadhyay A, Steely CE. A higher order theory for modeling composite laminates with induced strain actuators. Composites Part B 1997;28B:243–52.
[21] Chen CQ, Shen YP. Optimal control of active structures with piezoelectric modal sensors and actuators. Smart Mater Struct 1997;6:403–9.
[22] Chen CQ, Wang XM, Shen YP. Finite element approach of vibration control using self-sensing piezoelectric actuators. Comput Struct 1996;60:505–13.
[23] Chen SH, Wang ZD, Liu XH. Active vibration control and suppression for intelligent structures. J Sound Vibr 1997;20:167–77.
[24] Chen T, Baz A. Performance characteristics of active constrained layer damping versus passive constrained layer damping with active control. In: Varadan VV, Chandra J, editors. Smart Struct Mater. Washington: SPIE, 1996; 2715:256–269.
[25] Chin LC, Varadan VV, Varadan VK. Hybrid finite element formulation for periodic piezoelectric arrays subjected to fluid loading. Int J Numer Methods Engrg 1994;37:2987–3003.
[26] Crawley EF. Intelligent structures for aerospace: a technology overview and assessment. AIAA J 1994;32:1689–99.
[27] de Faria AR, de Almeida SFM. Modeling of actively

[1] The reference numbers 28, 37, 113 indicate papers to which the author had no access (not included in the discussion).

damped beams with piezoelectric actuators with finite stiffness bond. J Intell Mater Syst Struct 1996;7:677–88.

[28] Detwiler DT, Shen MH, Venkayya VB. Finite element analysis of laminated composite structures containing distributed piezoelectric actuators and sensors. Finite Elem Anal Des 1995;20:87–100.

[29] Di Sciuva M, Icardi U, Villani M. Finite element analysis of composite plates with induced-strain actuators: an evaluation of smeared and discrete-layer models. In: Santini P, Rogers CA, Murotsu Y, editors. Seventh Int. Conf. Adaptive Struct. Basel: Technomic, 1997. pp. 435–48.

[30] Gaudenzi P, Bathe KJ. An iterative finite element procedure for the analysis of piezoelectric continua. J Intell Mater Syst Struct 1995;6:266–73.

[31] Ghandi K, Hagood NW. Nonlinear finite element modeing of phase transitions in electro-mechanically coupled material. In: Varadan VV, Chandra J, editors. Smart Struct Mater. Washington: SPIE 1996;2715:121–140.

[32] Ghandi K, Hagood NW. A hybrid finite element model for phase transition in nonlinear electro-mechanically coupled material. In: Varadan VV, Chandra J, editors. Smart Struct Mater. Washington: SPIE 1997;3039:97–112.

[33] Ghiringhelli GL, Mantegazza P, Masarati P. Numerical modelling of anisotropic non homogeneous beams with embedded piezoelectric sensors and actuators. In: Santini P, Rogers CA, Murotsu Y, editors. Seventh Int. Conf. Adaptive Struct. Basel: Technomic, 1997. pp. 24–37.

[34] Gilewsky W, Radwanska M. Survey of finite element models for the analysis of moderately thick shells. Finite Elem Anal Des 1991;9:1–21.

[35] Guo N, Cawley P, Hitchings D. The finite element analysis of the vibration characteristics of piezoelectric discs. J Sound Vibr 1992;159:115–38.

[36] Ha KH. Finite element analysis of sandwich plates: an overview. Comput Struct 1990;37:397–403.

[37] Ha SK, Keilers C, Chang FK. Analysis of laminated composites containing distributed piezoelectric ceramics. J Intell Mater Syst Struct 1990;2:59–71.

[38] Ha SK, Keilers C, Chang FK. Finite element analysis of composite structures containing distributed piezoelectric sensors and actuators. AIAA J 1992;30:772–80.

[39] Han JH, Lee I. Active damping enhancement of composite plates with electrode designed piezoelectric materials. J Intell Mater Syst Struct 1997;8:249–59.

[40] Hanagud SM, Obal W, Calise AJ. Optimal vibration control by the use of piezoelectric sensors and actuators. J Guidance, Control Dyn 1992;15:1199–206.

[41] Heyliger PR, Ramirez G, Saravanos DA. Coupled discrete-layer finite elements for laminated piezoelectric plates. Comm Num Methods Engrg 1994;10:971–81.

[42] Hill LR, Farris TN. Three-dimensional piezoelectric boundary elements. In: Varadan VV, Chandra J, editors. Smart Struct Mater. Washington: SPIE 1997;3039:406–417.

[43] Hom CL, Shankar N. A finite element method for electrostrictive ceramics. Int J Solids Struct 1996;33:1757–79.

[44] Hom CL, Shankar N. A numerical analysis of relaxor ferroelectric multi-layered actuators and 2–2 composite arrays. Smart Mater Struct 1995;4:305–17.

[45] Hwang WS, Hwang W, Park HC. Vibration control of laminated composite plate with piezoelectric sensor/actuator: active and passive control methods. Mech Syst Signal Process 1994;8:571–83.

[46] Hwang WS, Park HC, Ha SK. Finite element modelling of piezoelectric sensors and actuators. AIAA J 1993;31:930–7.

[47] Hwang WS, Park HC, Hwang W. Vibration control of laminated composite plate with piezoelectric sensor/actuator: finite element formulation and modal analysis. J Intell Mater Syst Struct 1993;4:317–29.

[48] Islam AS, Craig KC. Damage detection in composite structures using piezoelectric materials. Smart Mater Struct 1994;3:318–28.

[49] Kagawa Y. A new approach to analysis and design of electro-mechanical problems by finite element technique. J Acoust Soc Am 1971;49:1348–56.

[50] Kagawa Y, Yamabushi T. Finite element approach for piezoelectric circular rod. IEEE Trans Sonics Ultrasonics 1976;SU-23:379–84.

[51] Kang YK, Kim MH, Park HC. Multi-modal vibration control of laminated composite plates using piezoelectric materials. In: Gobin PF, Tatibourt J, editors. Third Int Conf Intelligent Mater, vol. 2779. Washington: SPIE, 1996. pp. 688–93.

[52] Kang YK, Park HC, Hwang W, Han KS. Prediction and measurement of modal damping of laminated composite beams with piezoceramic sensor/actuator. J Intell Mater Syst Struct 1996;7:25–32.

[53] Kannan KS, Dasgupta A. A non linear Galerkin finite element theory for modeling magnetostrictive smart structures. Smart Mater Struct 1997;6:341–50.

[54] Kim J, Varadan VV, Varadan VK. Finite element optimization methods for the active control of radiated sound from a plate structure. Smart Mater Struct 1995;4:318–26.

[55] Kim J, Varadan VV, Varadan VK, Bao XQ. Finite element modeling of a smart cantilever plate and comparison with experiments. Smart Mater Struct 1996;5:165–70.

[56] Kim J, Varadan VV, Varadan VK. Finite element modeling of structures including piezoelectric active devices. Int J Numer Methods Engrg 1997;40:817–32.

[57] Kim J, Varadan VV, Ko B. Finite element modeling of active cabin noise control problems. In: Varadan VV, Chandra J, editors. Smart Struct Mater. Washington: SPIE 1997;3039:305–314.

[58] Koko IS, Orisamolu IR, Smith MJ, Alepan UO. Finite element based design tool for smart composite structures. In: Varadan VV, Chandra J, editors. Smart Struct Mater. Washington: SPIE 1997;3039:125–134.

[59] Lam KY, Peng XQ, Liu GR, Reddy JN. A finite element model for piezoelectric composite laminates. Smart Mater Struct 1997;6:583–91.

[60] Lammering R. The application of a finite shell element for composites containing piezoelectric polymers in vibration control. Comput Struct 1991;41:1101–9.

[61] Lammering R. The application of a finite shell element

for composites with integrated piezoelectric polymers. Zamm 1992;72:216–20.
[62] Lee HJ, Saravanos DA. Generalized finite element formulation for smart multilayered thermal piezoelectric composite plates. Int J Solids Struct 1997;34:3355–71.
[63] Lerch R. Simulation of piezoelectric devices by two and three-dimensional finite elements. IEEE Trans Ultrason Freq Control 1990;UFC-37:233–47.
[64] Lesieutre G, Lee U. A finite element for beams having segmented active constrained layers with frequency-dependent viscoelastics. Smart Mater Struct 1996;5:615–27.
[65] Lim YH, Varadan VV, Varadan VK. Closed loop finite element modeling of active structural damping in the frequency domain. Smart Mater Struct 1997;6:161–8.
[66] Lim YH, Varadan VV, Varadan VK. Finite element modeling of the transient response of MEMs sensors. Smart Mater Struct 1997;6:53–61.
[67] Lin MW, Abatan AO, Rogers CA. Application of commercial finite element codes for the analysis of induced strain-actuated structures. J Intell Mater Syst Struct 1994;5:869–75.
[68] Liu GR, Yili Z, Lam KY, Peng XQ, Tani J. Finite element modeling of piezoelectric sensors and actuators bonded in thick composite laminates. In: Murotsu Y et al., editors. Eighth Int. Conf. Adaptive Struct. Technologies. Basel: Technomic, 1997. pp. 113–22.
[69] Mackerle J. Finite element linear and nonlinear static and dynamic analysis of structural elements: a bibliography (1992–1995). Engng Computations 1997;14:347–440.
[70] Naillon M, Coursant RH, Besnier F. Analyse des structures piezoelectrique par une methode d'element finis. Acta Electronica 1983;25:341–62.
[71] Noor AK. Bibliography of books and monographs on finite element technology. Appl Mech Rev 1991;44:307–17.
[72] Panahandah M, Cui Y, Kaspe EP. Simulation of coupled systems in smart structures. In: Varadan VV, Chandra J, editors. Smart Struct Mater. Washington: SPIE 1997;3039:292–304.
[73] Prakah-Asante KO, Craig KC. The application of multi-channel design methods for vibration control of active structure. Smart Mater Struct 1994;3:329–43.
[74] Rahmoune M, Osmont D, Benjeddou A, Ohayon R. Finite element modeling of a smart structure plate system. In: Santini P et al., editors. Seventh Int. Conf. Adapt. Struct. Basel: Technomic, 1997. pp. 463–73.
[75] Rajapakse RKND. Boundary element method for piezoelectric solids. In: Varadan VV, Chandra J, editors. Smart Struct Mater Washington: SPIE 1997;3039:418–427.
[76] Rao SS, Sunar M. Analysis of distributed thermopiezoelectric sensors and actuators in advanced intelligent structures. AIAA J 1993;31:1280–6.
[77] Rao SS, Sunar M. Piezoelectricity and its use in the disturbance sensing and control of flexible structures: a survey. Appl Mech Rev 1994;47:113–23.
[78] Ray MC, Bhattacharyya R, Samanta B. Static analysis of an intelligent structure by the finite element method. Comput Struct 1994;52:617–31.
[79] Reddy JN. Finite element modeling of layered, anisotropic composite plates and shells: a review of recent research. Schock Vibr Dig 1981;13:3–12.
[80] Ro J, Al-Ali A, Baz A. Control of sound radiation from a fluid loaded plate using active constrained layer damping. In: Proc. 6th Int. Conf. Recent Adva. Structural Des. ISVR. 1997. pp. 1257–73.
[81] Ro J, El-Din KS, Baz A. Vibration control of tubes with internally moving loads using active constrained layer damping. In: Clark WW et al., editors. Active/Passive Vibr. Control Nonlinear Dynamics Struct. ASME 1997;DE-95/AMD-223:1–11.
[82] Robbins DH, Reddy JN. Analysis of piezoelectrically actuated beams using a layer-wise displacement theory. Comput Struct 1991;41:265–79.
[83] Saravanos DA, Heyliger PR. Coupled layerwise analysis of composite beams with embedded piezoelectric sensors and actuators. J Intell Mater Syst Struct 1995;6:350–63.
[84] Saravanos DA, Heyliger PR, Hopkins DA. Layerwise mechanics and finite element for the dynamic analysis of piezoelectric composite plates. Int J solids Struct 1997;34:359–78.
[85] Seshu P, Naganathan NG. Finite element analysis of strain transfer in an induced strain actuator. Smart Mater Struct 1997;6:76–88.
[86] Shah DK, Joshi SP, Chan WS. Static structural response of plates within piezoceramic layers. Smart Mater Struct 1993;2:172–80.
[87] Shen MHH. Analysis of beams containing piezoelectric sensors and actuators. Smart Mater Struct 1994;3:439–47.
[88] Shen MHH. A new modeling technique for piezoelectrically actuated beams. Comput Struct 1995;57:361–6.
[89] Shen JY, Sharpe L, Jr. A finite element model of the aeroelasticity analysis of hypersonic panels. Part III: Flutter suppression. In: Varadan VV, Chandra J editors. Smart Struct Mater. Washington: SPIE 1997;3039:315–323.
[90] Shields W, Ro J, Baz A. Control of sound radiation from a plate into an acoustic cavity using active piezoelectric damping composites. In: Varadan VV, Chandra J, editors. Smart Struct Mater. Washington: SPIE 1997;3039:70–90.
[91] Shieh RC. Governing equations and finite element models for multiaxial piezoelectric beam sensors/actuators. AIAA J 1994;32:1250–8.
[92] Smyser CP, Chandrashekhara K. Robust vibration control of composite beams using piezoelectric devices and neural networks. Smart Mater Struct 1997;6:178–89.
[93] Suleman A, Venkayya VB. A simple finite element formulation for a laminated composite plate with piezoelectric layers. J Intell Mater Syst Struct 1995;6:776–82.
[94] Surace G, Cardascia L, Anghel V. Finite element formulation for piezoelectrically actuated Timoshenko beam. In: Santini P, Rogers CA, Morotsu Y, editors. Seventh Int. Conf. Adapt. Struct. Basel: Technomic, 1997. pp. 498–507.
[95] Thirupathi SR, Seshu P, Naganathan NG. A finite static analysis of smart turbine blades. Smart Mater Struct 1997;6:607–15.

[96] Trindade MA, Benjeddou A, Ohayon R. Performances d'un mecanisme hybride actif/passif d'amortissement des vibrations. 4e Coll. GDR Vibroacoustique-CNRS 1138, Marseille: LMA 1998, in press.

[97] Trindade MA, Benjeddou A, Ohayon R. Parametric analysis of the vibration control of sandwich beams through shear-based piezoelectric actuation. J Inell Mater Syst Struct, in review.

[98] Tzou HS. Integrated distributed sensing and active vibration suppression of flexible manipulators using distributed piezoelectrics. J Robotic Syst 1989;6:745–67.

[99] Tzou HS, Tseng CI. Distributed piezoelectric sensor/actuator design for dynamic measurement/control of distributed parameter systems:A piezoelectric finite element approach. J Sound Vibr 1990;138:17–34.

[100] Tzou HS, Tseng CI. Distributed vibration control and identification of coupled elastic-piezoelectric systems:Finite element formulation and application. Mech Syst Signal Process 1991;5:215–31.

[101] Tzou HS, Tseng CI. Distributed modal identification and vibration control of continua: Piezoelectric finite element formulation and analysis. J Dyn Syst, Measurement Control 1991;113:500–5.

[102] Tzou HS, Tseng CI, Bahrami H. A piezoelectric hexahedron finite element applied to design of smart continua. Finite Elem Analy Des 1994;16:27–42.

[103] Tzou HS, Tseng CI, Wan GC. Distributed structural dynamics control of flexible manupulators-II: Distributed sensor and active electromechanical actuator. Comput Struct 1990;35:679–87.

[104] Tzou HS, Ye R. Piezothermoelasticity and precision control of piezoelectric laminates: theory and finite element analysis. J Vibr Acoust 1994;116:489–95.

[105] Tzou HS, Ye R. Analysis of piezoelectric systems with laminated piezoelectric triangular shell elements. AIAA J 1996;34:110–5.

[106] van Nostrand W., Knowles GJ, Inman D. Finite element model for active constrained-layer damping. In: Anderson GL, Lagoudas DC, editors. Active Mater Smart Struct. Washington: SPIE 1994;2427:124–139.

[107] Varadan VV, Chin L, Varadan VK. Finite element modeling of flextensional electroacoustic transducers. Smart Mater Struct 1993;2:201–7.

[108] Varadan VV, Lim YH, Varadan VK. Closed loop finite element modeling of active/passive damping in structural vibration control. Smart Mater Struct 1996;5:685–94.

[109] Veley DE, Rao SS. Optimal design of structures with active constrained layer damping. In: Johnson C, editor. Smart Struct Mater. Wasington: SPIE 1995;2445:98–109.

[110] Veley DE, Rao SS. A comparison of active, passive and hybrid damping treatments in structural design. Smart Struct Mater 1996;5:660–71.

[111] Yang HTY, Saigal S, Liaw DG. Advances of thin shell finite elements and some applications — version I. Comput Struct 1990;35:481–504.

[112] Yin L, Shen Y. Strain sensing of composite plates subjected to low velocity impact with distributed piezoelectric sensors: a mixed finite element approach. J Sound Vibr 1997;199:17–31.

[113] Zako M, Tsujikami T. Three-dimensional intelligent finite element method for design of composite structures — design of symmetric laminates by shell element. J Soc Mater Sci Jpn 1995;44:916–20.