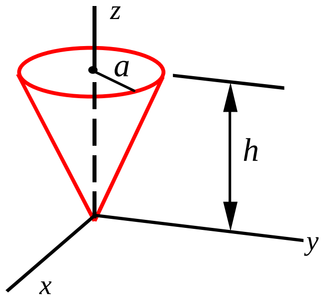
MASS MOMENT OF INERTIA

Problem 1:



Calculate the mass moment of inertia of the cone about the *z*-axis. Assume the cone is made of a uniform material of density ρ (mass per unit volume).

Solution:

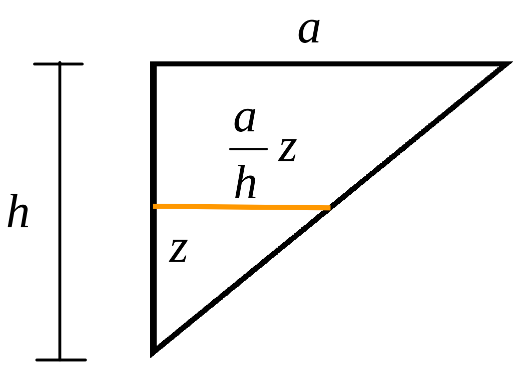
The mass moment of inertia about the *z*-axis is given by

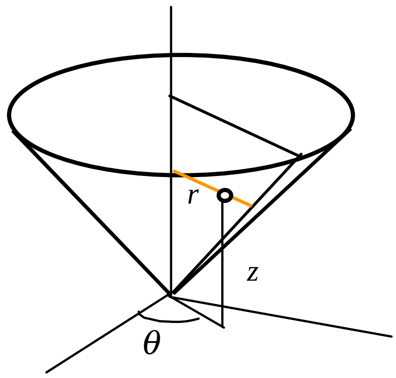
$$I_{zz} = \int_B r^2 dm = \int_V r^2 \rho \, dV$$

The element of volume in a cylindrical coordinate system is given by

$$dV = r dr d\theta \, dz$$

The domain of the cone in cylindrical coordinates is defined by





$$\begin{cases} 0 \le z \le h \\ 0 \le r \le \dfrac{a}{h} z \\ 0 \le \theta \le 2\pi \end{cases}$$

Therefore, the mass moment of inertia about the *z*-axis can be written as

$$I_{zz} = \int_V r^2 \rho \, dV = \int_{z=0}^{z=h} \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\frac{a}{h}z} r^3 \rho \, dr d\theta \, dz$$

$$= \int_{z=0}^{z=h} \int_{\theta=0}^{\theta=2\pi} \frac{a^4}{4h^4} z^4 \rho \, d\theta \, dz$$

$$= \int_{z=0}^{z=h} \frac{\pi a^4}{2h^4} z^4 \rho \, dz = \frac{\pi a^4 h \rho}{10}$$

For a uniform cone the density can be calculated using the total mass and total volume of the cone so that

$$\rho = \frac{m}{V} = \frac{m}{\frac{1}{3}\pi a^2 h}$$

Therefore, the moment of inertia in terms of the total mass of the cone can be written as

$$I_{zz} = \frac{\pi a^4 h \rho}{10} = \frac{3a^2 m}{10}$$