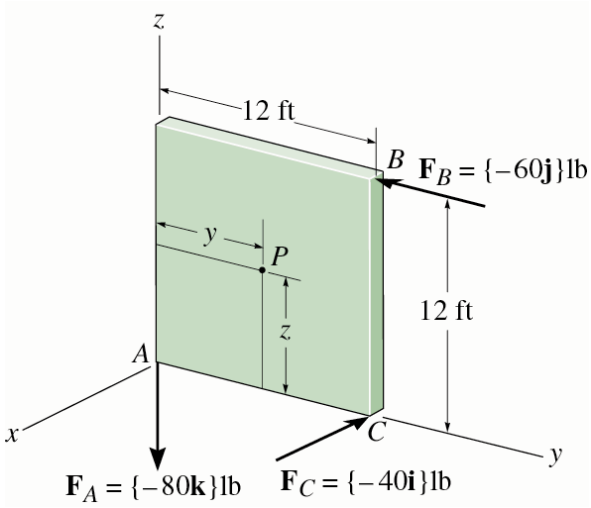
Moments 7:

Problem 1:

Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point P(y,z) where its line of action intersects the plate.



Solution:

Resultant Force Vector :

$$\mathbf{F_R} = \{-40\,\mathbf{i} - 60\,\mathbf{j} - 80\,\mathbf{k}\}\text{lb}$$

$$F_R = \sqrt{(-40)^2 + (-60)^2 + (-80)^2} = 107.7\text{ lb}$$

$$\mathbf{u}_{\mathbf{F_R}} = \frac{\mathbf{F_R}}{F_R} = \frac{\{-40\,\mathbf{i} - 60\,\mathbf{j} - 80\,\mathbf{k}\}}{107.7}$$

$$\Rightarrow \mathbf{u}_{\mathbf{F_R}} = \{-0.3714\,\mathbf{i} - 0.5571\,\mathbf{j} - 0.7428\,\mathbf{k}\}\text{lb}$$

Resultant Moment : The line of action of $\mathbf{M_R}$ of the wrench is parallel to the line of action of $\mathbf{F_R}$. Assume that both $\mathbf{F_R}$ and $\mathbf{M_R}$ have the same sense.

$$\Rightarrow \mathbf{u}_{\mathbf{M_R}} = \{-0.3714\,\mathbf{i} - 0.5571\,\mathbf{j} - 0.7428\,\mathbf{k}\}\text{lb}$$

$$\mathbf{u}_{\mathbf{M_R}} = \{-0.3714\,\mathbf{i} - 0.5571\,\mathbf{j} - 0.7428\,\mathbf{k}\}\text{lb}$$

Let x', y', z' be the axes with origin P

$$(\mathrm{M_R})_{x'} = \sum M_{x'}$$

$$\Rightarrow -0.3714 M_R = 60(12 - z) + 80y \qquad (1)$$

$$(\mathrm{M_R})_{y'} = \sum M_{y'}$$

$$\Rightarrow -0.5571 M_R = 40z \qquad (2)$$

$$(\mathrm{M_R})_{z'} = \sum M_{z'}$$

$$\Rightarrow -0.7428 M_R = 40(12 - y) \qquad (3)$$

$$-0.3714M_R = 60(12 - z) + 80y \quad (1)$$
$$-0.5571M_R = 40z \quad (2)$$
$$-0.7428M_R = 40(12 - y) \quad (3)$$

Solve the system of 3 equations :

$$(2) \Rightarrow M_R = -71.8z$$

Substitute (2) in (1) and (3)

$$\Rightarrow \begin{cases} -0.3714(-71.8z) = 60(12 - z) + 80y \\ -0.7428(-71.8z) = 40(12 - y) \end{cases}$$

$$\Rightarrow \begin{cases} 86.67z = 720 + 80y \\ 53.33z = 480 - 40y \end{cases}$$

$$\Rightarrow \begin{cases} z = 8.31 + 0.92y \quad (4) \\ z = 9 - 0.75y \quad (5) \end{cases}$$

$$M_R = -71.8z \quad (2)$$

$$\begin{cases} z = 8.31 + 0.92y \quad (4) \\ z = 9 - 0.75y \quad (5) \end{cases}$$

$$(4) \text{ in } (5) \Rightarrow 8.31 + 0.92y = 9 - 0.75y$$
$$\Rightarrow y = 0.41 \text{ ft}$$

Substitute $y = 0.41$ ft in equation (5)

$$\Rightarrow z = 9 - 0.75(0.41) = 8.69 \text{ ft}$$

$$\Rightarrow \text{P}(0.41, 8.69) \text{ ft}$$

$$M_R = -71.8z \quad (2)$$

P(0.41, 8.69) ft

$$M_R = -71.8z$$
$$\Rightarrow M_R = -71.8(8.69) = -624 \text{ lb.ft}$$
$$\Rightarrow M_R = -624 \text{ lb.ft at P}(0.41, 8.69) \text{ ft}$$

The negative sign indicates that the line of action of $\mathbf{M_R}$ is directed in the opposite sense to that of $\mathbf{F_R}$.