Robust shape optimization method for shell structures with unknown loadings

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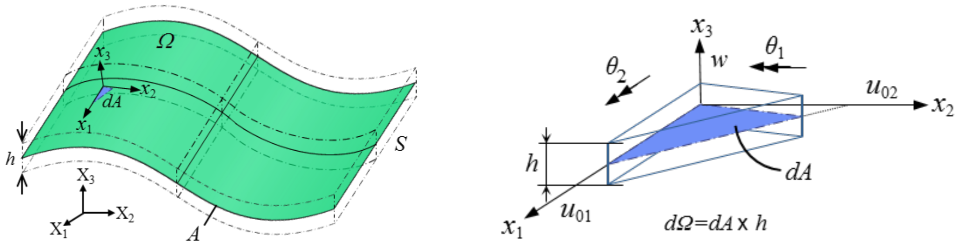
1. Abstract
In this paper, we propose a robust shape optimization method for a shell structure with unknown loadings. The concept of the principal compliance minimization for minimizing the maximal compliance is applied to the shape optimization design of a shell structure. The principal compliance minimization problem can be transformed to the equivalent maximization problem of the fundamental eigenvalue of the stiffness term, and this problem is formulated as the distributed-parameter optimization problem based on the variational method. The derived shape gradient function is applied to the H^1 gradient method for shells to determine the optimal shape variation, or the optimal free-form. With this method, the optimal smooth curvature distribution of a shell structure can be determined without shape parameterization. The calculated results show the effectiveness of the proposed method for robust shape optimization of a shell with unknown loadings.
2. Keywords: Robust shape optimization, shell structure, loading uncertainties, principal compliance, H^1 gradient Method,

3. Introduction
Structural optimization techniques are widely utilized in many structural design fields. In general optimum design problems, the boundary condition is treated deterministically, although the condition such as loading condition frequently contains uncertainties. A design problem we often encounter loading conditions from all directions or multiple loading conditions by sharing of parts in actual design problems, which is one of the design problems with unknown or uncertain loadings. As the optimal design is generally vulnerable to the variation of loading because the structural performances such as stiffness or strength are strongly influenced by loading, the reliability design is often introduced to the formulation of optimal design problems. Safety factor or probabilistic approach is a method of reliability design problems. However, too large factor often causes excessive performances, redundant structure and weight gain.

Another approach to avoid the vulnerability of the optimally designed structure to variations of loading has been proposed by Cherkaev et al. [1], in which the concept of the principal compliance minimization is introduced, which is defined as the minimization of the maximal compliance under the worst possible loading. They formulated it as a min-max compliance problem, and showed that the principal compliance minimization problem can be transformed to the equivalent maximization problem of the fundamental eigenvalue of the stiffness term. They applied this idea to a simple size optimization problem. Takezawa et al. [2] applied the concept of the principle compliance to a topology optimization problem under the assumption that the loading domain is limited in a small sub-domain of the linear elastic domain to solve the full size linear elastic system efficiently.

In this paper, we newly propose a robust shape optimization method for shell structures by employing this concept to the free-form optimization method for shells. This method is a parameter-free shape optimization method based on the variational method, which was proposed by one of the authors [3]. In this method, a shape optimization problem is formulated in the continuous system, and the optimal smooth curvature distribution of a shell structure is determined without any shape parameterization, although almost all shape optimization methods need shape parameterization. The principal compliance minimization problem is transformed to the equivalent maximization problem of the fundamental eigenvalue of the stiffness term based on this concept. The transformed objective functional is maximized under the volume constraint, and the shape gradient function is theoretically derived using the material derivative method and the adjoint variable method. The derived shape gradient function is applied to the H^1 gradient method for shells to determine the optimal shape variation, or the optimal free-form. We carried out a numerical example to verify the effectiveness of the proposed robust shape optimization method.

4. Governing equation for a shell as a set of infinitesimal flat surfaces



(a) Geometry of shell (b) Local coordinates and DOF of surface.

Fig.1 Shell consisting of infinitesimal flat surfaces.

As shown in Fig.1 and Eq.(1), consider a linear elastic shell having an initial bounded domain $\Omega \subset \mathbb{R}^3$, mid-area A with the boundary of ∂A, side surface S and thickness h. It is assumed that a shell structure occupying a bounded domain is a set of infinitesimal flat surfaces as shown in Fig.1, and stress and strain of the shell are expressed by superposing the membrane and bending components based on the Reissner-Mindlin theory.

$$\Omega = \left\{ (x_1,x_2,x_3) \in \mathbb{R}^3 \;\middle|\; (x_1,x_2) \in A \subset \mathbb{R}^2, x_3 \in \left(-\frac{h}{2},\frac{h}{2}\right) \right\}, \Omega = A\times\left(-\frac{h}{2},\frac{h}{2}\right), \quad S = \partial A \times\left(-\frac{h}{2},\frac{h}{2}\right) \tag{1}$$

As shown in Fig.1(b), Eq.(2) and Eq.(3), the displacement vector expressed by the displacements in local coordinate $\boldsymbol{u} = \{u_i\}_{i=1,2,3}$ is considered by dividing into the in-plane direction $\{u_\alpha\}_{\alpha=1,2}$ and the out-of-plane direction u_3. In this paper, the subscripts of the Greek letters are expressed as $\alpha = 1,2$ and the tensor subscript notation uses Einsteins summation convention and a partial differential notation.

$$u_\alpha(x_1,x_2,x_3) = u_{0\alpha}(x_1,x_2) - x_3\theta_\alpha(x_1,x_2) \tag{2}$$

$$u_3(x_1,x_2,x_3) = w(x_1,x_2) \tag{3}$$

where $\boldsymbol{u}_0 = \{u_{0\alpha}\}_{\alpha=1,2}$, w and $\boldsymbol{\theta} = \{\theta_\alpha\}_{\alpha=1,2}$ express the in-plane displacements, out-of-plane displacement and rotational angles of the mid-area of the shell, respectively. Then, the weak form state equation relative to $\boldsymbol{u} = (\boldsymbol{u}_0, w, \boldsymbol{\theta}) \in U$ can be expressed as Eq.(4). An in-plane load $\{f_\alpha\}_{\alpha=1,2}$, an out-of-plane load $\{f_3\}$ are considered as the external forces.

$$a\left((\boldsymbol{u}_0,w,\boldsymbol{\theta}),(\bar{\boldsymbol{u}}_0,\bar{w},\bar{\boldsymbol{\theta}})\right) = l\left((\bar{\boldsymbol{u}}_0,\bar{w},\bar{\boldsymbol{\theta}})\right), \quad \forall(\bar{u}_0,\bar{w},\bar{\boldsymbol{\theta}}) \in U, \quad (u_0,w,\boldsymbol{\theta}) \in U \tag{4}$$

where $(\bar{\ })$ expresses a variation. In addition, the bilinear form $a(\cdot,\cdot)$ and the linear form $l(\cdot)$ are defined.

$$a\left((\boldsymbol{u}_0,w,\boldsymbol{\theta}),(\bar{\boldsymbol{u}}_0,\bar{w},\bar{\boldsymbol{\theta}})\right) = \int_\Omega \left\{ C_{\alpha\beta\gamma\delta}\left(u_{0\alpha,\beta} - x_3\theta_{\alpha,\beta}\right)\left(\bar{u}_{0\gamma,\delta} - x_3\bar{\theta}_{\gamma,\delta}\right) + C^s_{\alpha\beta}\left(w_{,\alpha} - \theta_\alpha\right)\left(\bar{w}_{,\alpha} - \bar{\theta}_\alpha\right) \right\} d\Omega \tag{5}$$

$$l\left((\bar{\boldsymbol{u}}_0,\bar{w},\bar{\boldsymbol{\theta}})\right) = \int_\Omega \bar{f}_i\,\bar{u}_i\, d\Omega = \int_\Omega \left\{ f_1\left(\bar{u}_{01} - x_3\bar{\theta}_1\right) + f_2\left(\bar{u}_{02} - x_3\bar{\theta}_2\right) + f_3\, w \right\} d\Omega \tag{6}$$

where $\left\{C_{\alpha\beta\gamma\delta}\right\}_{\alpha,\beta,\gamma,\delta=1,2}$ and $\left\{C^s_{\alpha\beta}\right\}_{\alpha,\beta=1,2}$ express an elastic tensor with respect to the membrane stress and an elastic tensor with respect to the shearing stress, respectively. It will be noted that U in Eq.(4) is given by the following equation.

$$U = \left\{ \left(u^{(1)}_{0,1}, u^{(1)}_{0,2}, w^{(1)}, \theta^{(1)}_1, \theta^{(1)}_2\right) \in \left(\mathrm{H}^1(A)\right)^5 \mid \text{satisfy the given Dirichlet condition on each sub-boundary} \right\} \tag{7}$$

where H^1 is the Sobolev space of order 1.

5. Robust shape optimization of shell structure

5.1. Domain variation

We consider that a linear elastic shell structure having an initial domain Ω, mid-area A, boundary ∂A and side surface S undergoes domain variation $\boldsymbol{V}$ (i.e., design velocity field) in the out-of-plane direction such that its domain, mid-area, boundary and side surface become Ω_s, A_s, ∂A_s ,and S_s as shown in Fig.2, respectively. It is assumed that the thickness h remains constant under the domain variation. The subscript s expresses the iteration history of the domain variation.

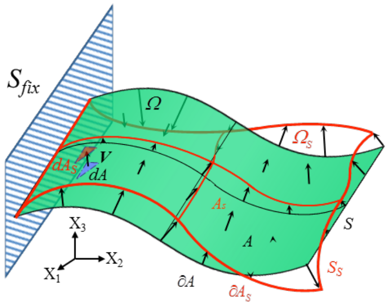


Fig.2 Out-of-plane shape variation by $\boldsymbol{V}$.

5.2. Principal compliance

In a domain Ω, the principal compliance l_p of a structure is defined as the maximal compliance under all admissible unknown loadings [1].

$$l_p = \max(l) = \max\left\{\int_A f_i\, u_i\, dA\right\} \tag{8}$$

where the unknown loadings satisfy following normalizing condition as shown in Eq.(9).

$$\frac{1}{2}\int_\Omega A_{ij}^{-1} f_i\, f_j\, dx = 1 \tag{9}$$

where $\boldsymbol{f}$ is the force vector, and $\{A_{ij}\}_{i,j=1,2,3}$ is a diagonal tensor which expresses the loading positions and their magnitude. The component of A_{ij} is proportional to the magnitude of the force corresponding, and has an infinitesimal δ at the point without loading. Then, the inverse tensor of A_{ij} expresses the weight factor to describe the set of admissible loadings. The principal compliance minimization problem can be transformed to the equivalent maximization problem of the fundamental eigenvalue $\lambda^{(1)}$, because the maximal compliance can be expressed as the inverse of the fundamental eigenvalue of the stiffness tensor [1].

$$l_p \propto 1/\lambda^{(1)} \tag{10}$$

$$a((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \lambda^{(1)} b((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})), \quad (\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}) \in U,\ \forall(\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}) \in U \tag{11}$$

where the bilinear form $b(\cdot,\cdot)$ is defined as

$$b((\boldsymbol{u}_0, w, \boldsymbol{\theta})) = \int_\Omega A_{ij} u_i \bar{u}_j dx = \int_\Omega \left\{A_{\alpha\beta}\left(u_{0\alpha} - x_3\theta_\alpha\right)\left(\bar{u}_{0\beta} - x_3\bar{\theta}_\beta\right) + A_{33} w\bar{w}\right\} dx \tag{12}$$

5.3. Problem formulation

Let us consider a free-form optimization problem for minimizing the principal compliance of a shell structure. Letting the state equations in Eq.(11) and the volume be the constraint conditions, and the fundamental eigenvalue be the objective functional to be minimized, a distributed parameter shape optimization problem for determining the optimal design velocity field $\boldsymbol{V}$ can be formulated based on the variational method as

Given	$A, \hat{M}$	(13)
Find	$\boldsymbol{V}$	(14)
that minimize	$-\lambda^{(1)}$	(15)
subject to	Eq.(11) and $M(=\int_A h dA) \le \hat{M}$	(16)

where M and $\hat{M}$ denote the volume and its constraint value, respectively.

5.4. Derivation of shape gradient function and optimality conditions

The Lagrange multiplier method is used to transform this constrained shape optimization problem to the unconstrained one. Letting $(\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})$ and Λ denote the Lagrange multipliers for the state equation and volume constraints, respectively, the Lagrange functional L associated with this problem can be expressed as

$$\begin{aligned} L((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}), \Lambda) = -\lambda^{(1)} + \lambda^{(1)} b((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) \\ - a((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) + \Lambda(M - \hat{M}) \end{aligned} \tag{17}$$

The material derivative of the Lagrange functional L can be derived as shown in Eq.(18) using the design velocity field $\boldsymbol{V}$.

$$\begin{aligned} \dot{L} = & -a((\boldsymbol{u}_0^{(1)\prime}, w^{(1)\prime}, \boldsymbol{\theta}^{(1)\prime}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) + \lambda^{(1)} b((\boldsymbol{u}_0^{(1)\prime}, w^{(1)\prime}, \boldsymbol{\theta}^{(1)\prime}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) - a((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0', \bar{w}', \bar{\boldsymbol{\theta}}')) \\ & + \lambda^{(1)} b((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0', \bar{w}', \bar{\boldsymbol{\theta}}')) + \dot{\lambda}^{(1)} \{ b((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) - 1 \} + \Lambda'(M - \hat{M}) \\ & + \langle G\boldsymbol{n}, \boldsymbol{V} \rangle, \qquad \boldsymbol{V} \in C_\Theta \end{aligned} \tag{18}$$

$$\langle G\boldsymbol{n}, \boldsymbol{V} \rangle \equiv \int_A G\boldsymbol{n} \cdot \boldsymbol{V} dA = \int_A \left(G^{(1)} + G_M \right) \boldsymbol{V}_n dA \tag{19}$$

where $G\boldsymbol{n} (= \boldsymbol{G})$ expresses the shape gradient function. C_Θ is the suitably smooth function space that satisfies the constraints of the domain variation. H is twice the mean curvature of mid-area A . The notations $\boldsymbol{n}^{top}$ and $\boldsymbol{n}^{btm}$ denote unit outward normal vectors at the top surface and the bottom surface, respectively, and a unit normal vector at the mid-area $\boldsymbol{n}^{mid} (\equiv \boldsymbol{n}) = \boldsymbol{n}^{top} = -\boldsymbol{n}^{btm}$ is assumed by Shimoda et al. [3]. The coefficient function of the shape gradient function G consist of $G^{(1)}$ and G_M corresponding to 1st eigenvalue and volume constraint, respectively. The optimality conditions of the Lagrange functional L with respect to $(\boldsymbol{u}_0, w, \boldsymbol{\theta})$, $(\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})$ and Λ are expressed as

$$\begin{aligned} a((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0', \bar{w}', \bar{\boldsymbol{\theta}}')) = \lambda^{(1)} b((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0', \bar{w}', \bar{\boldsymbol{\theta}}')), \\ (\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}) \in U, \quad \forall (\bar{\boldsymbol{u}}_0', \bar{w}', \bar{\boldsymbol{\theta}}') \in U \end{aligned} \tag{20}$$

$$\begin{aligned} a((\boldsymbol{u}_0^{(1)\prime}, w^{(1)\prime}, \boldsymbol{\theta}^{(1)\prime}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = \lambda^{(1)} b((\boldsymbol{u}_0^{(1)\prime}, w^{(1)\prime}, \boldsymbol{\theta}^{(1)\prime}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})), \\ (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}) \in U, \quad \forall (\boldsymbol{u}_0^{(1)\prime}, w^{(1)\prime}, \boldsymbol{\theta}^{(1)\prime}) \in U, \end{aligned} \tag{21}$$

$$\lambda^{(1)} \{ b((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) = 1 \tag{22}$$

$$\Lambda(M - \hat{M}) = 0, \quad \Lambda \geq 0, \quad M - \hat{M} \leq 0 \tag{23}$$

When the optimality conditions are satisfied, L becomes

$$L = \langle G\boldsymbol{n}, \boldsymbol{V} \rangle \tag{24}$$

Considering the self-adjoint relationship $(\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}) = (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})$, which is obtained from comparing Eq.(20) and Eq.(21), the shape gradient functions $G^{(1)}, G_M$ are derived as

$$\begin{aligned} G = & -\{ C_{\alpha\beta\gamma\delta} (u_{0\alpha,\beta}^{(1)} + \frac{h}{2} \theta_{\alpha,\beta}^{(1)})(u_{0\gamma,\delta}^{(1)} + \frac{h}{2} \theta_{\gamma,\delta}^{(1)}) - C_{\alpha\beta\gamma\delta} (u_{0\alpha,\beta}^{(1)} - \frac{h}{2} \theta_{\alpha,\beta}^{(1)})(u_{0\gamma,\delta}^{(1)} - \frac{h}{2} \theta_{\gamma,\delta}^{(1)}) \} \\ & + \lambda^{(1)} \{ A_{\alpha\beta} (u_{0\alpha}^{(1)} + \frac{h}{2} \theta_\alpha^{(1)})(u_{0\beta}^{(1)} + \frac{h}{2} \theta_\beta^{(1)}) + A_{33} w^{(1)} w^{(1)} \} - \lambda^{(1)} \{ A_{\alpha\beta} (u_{0\alpha}^{(1)} - \frac{h}{2} \theta_\alpha^{(1)})(u_{0\beta}^{(1)} - \frac{h}{2} \theta_\beta^{(1)}) + A_{33} w^{(1)} w^{(1)} \} + hH\Lambda \end{aligned} \tag{25}$$

$$G_M = hH\Lambda \tag{26}$$

6. H^1 Gradient method for shells

The free-form optimization method for shell was proposed by Shimoda [3], which consists of main three processes; (1) Derivation of shape gradient function (2) Numerical calculation of shape gradient function (3) The H^1 gradient method for determining the optimal shape variation. The H^1 gradient method is a gradient method in a Hilbert space. The original H^1 gradient method was proposed by Azegami in 1994 [4] and also called the traction method. Shimoda modified the original method for free-form shell optimization. In the present paper, we employ the H^1 gradient method for shells to determine the optimal shape variation for the robust shape optimization problem. It is a node-based shape optimization method that can treat all nodes as design variables and does not require any design variable parameterization.

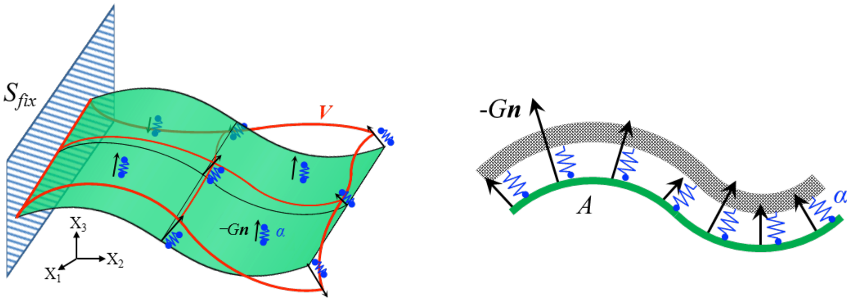


Fig.3 H^1 gradient method for shells.

This minimax problem may encounter the repeated eigenvalue problem of the objective functional. When this problem occurs, we change the objective and constraint functions as shown in Eq.(27). $r(\geq 2)$ denotes the number of repeating. The repeated eigenvalue is judged by introducing the range δ. In this paper we use $\delta\{^{+0.02\lambda^{(1)}}_{-0}$.

$$L((\boldsymbol{u}_0^{(1)}, w^{(1)}, \boldsymbol{\theta}^{(1)}), \quad ,(\boldsymbol{u}_0^{(r)}, w^{(r)}, \boldsymbol{\theta}^{(r)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}}), \Lambda)$$
$$= \sum_{k=1}^{r} \left\{ -\lambda^{(k)} + \lambda^{(k)} b((\boldsymbol{u}_0^{(k)}, w^{(k)}, \boldsymbol{\theta}^{(k)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) - a((\boldsymbol{u}_0^{(k)}, w^{(k)}, \boldsymbol{\theta}^{(k)}), (\bar{\boldsymbol{u}}_0, \bar{w}, \bar{\boldsymbol{\theta}})) \right\} + \Lambda(M - \hat{M}) \tag{27}$$

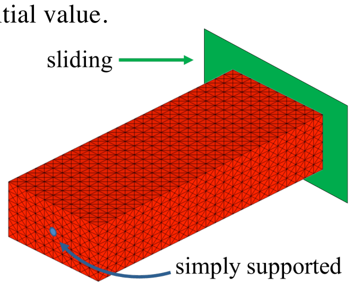
In an analogous way, the shape gradient function becomes

$$G = \sum_{k=1}^{r} \left(G^{(k)} \right) + G_M \tag{28}$$

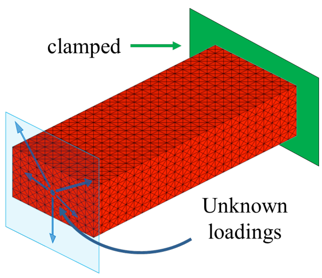
$$G = -\{C_{abgd}(u_{0a,b}^{(k)} + \frac{h}{2}q_{a,b}^{(k)})(u_{0g,d}^{(k)} + \frac{h}{2}q_{g,d}^{(k)}) - C_{abgd}(u_{0a,b}^{(k)} - \frac{h}{2}q_{a,b}^{(k)})(u_{0g,d}^{(k)} - \frac{h}{2}q_{g,d}^{(k)})\}$$
$$+ l^{(k)}\{A_{ab}(u_{0a}^{(k)} + \frac{h}{2}q_a^{(k)})(u_{0b}^{(k)} + \frac{h}{2}q_b^{(k)}) + A_{33}w^{(k)}w^{(k)}\} - l^{(k)}\{A_{ab}(u_{0a}^{(k)} - \frac{h}{2}q_a^{(k)})(u_{0b}^{(k)} - \frac{h}{2}q_b^{(k)}) + A_{33}w^{(k)}w^{(k)}\} + hH\Lambda \tag{29}$$

5. Results of numerical calculation

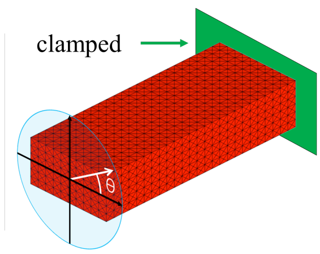
The proposing method is applied to a simple problem to confirm the validity of the proposed method. Fig.4 shows the shape optimization problem definition of a box-shaped cantilever shell structure. As shown in Fig.4(b), an unknown loading is applied to the center of the free end face. The volume constraint is set as the same as the initial value.



(a) Velocity analysis.



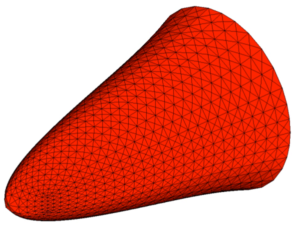
(b) Stiffness analysis.



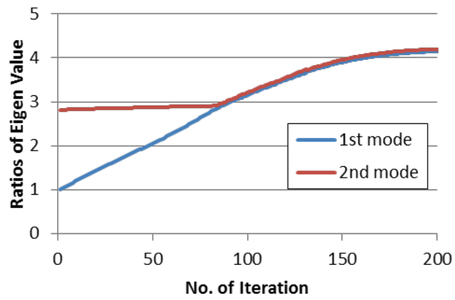
(c) Evaluation of compliance calculation.

Fig.4 Boundary conditions.

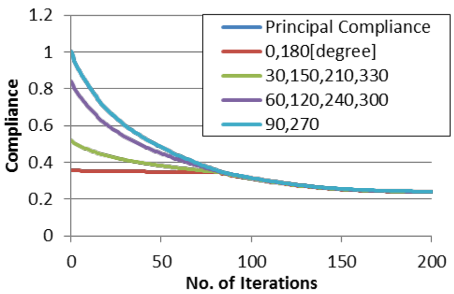
Fig.5(a) shows the obtained optimal shape. The clamped end expands while the area of free end narrows toward the loading point. The iteration convergence histories of the 1st and 2nd eigenvalues are shown in Fig.5(b). To confirm the robustness of the result, we use the polar coordinate in which the origin is the point of the loading, and measure the compliance of every 30 degree in the circumferential direction. The convergence history of the principal and the evaluation compliance of every 30 degree in the circumferential direction is shown in Fig.5(c), and comparison of compliances of each loading direction is shown in Fig.5(d). It is confirmed that the 1st eigenvalue is maximized as shown in Fig.5(b). The compliance is reduced by approximately 76% as shown in Fig.5(c)(d). We confirm that the optimal robust shape with high stiffness and independence of loading direction can be obtained with this method.



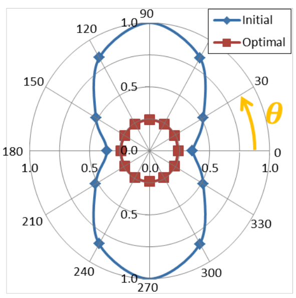
(a) Obtained shape



(b)Iteration histories of ratios of eigenvalue.



(c) Convergence history of principal and evaluation compliance.



(d) Comparison of compliances for loading directions.

Fig.5 Results of robust shape optimization.

5. Conclusions

In this paper, a robust shape optimization method for a shell structure with unknown loadings was constructed based on the concept of the principal compliance minimization which was transformed to the equivalent maximization problem of the fundamental eigenvalue of the stiffness term. A principal compliance maximization problem subject to both constraints of the volume and the state equation of shell structure was formulated as a distributed-parameter shape optimization problem, and the sensitivity function for this problem was theoretically derived. The derived shape gradient function was applied to the H^1 gradient method for shell structures to determine the robust optimal shape. The calculated result showed the effectiveness of the proposed method for robust shape optimization of a shell structure with unknown loading, or for creating the smooth optimal shell structure with high stiffness and independence of loading directions.

6. References

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