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Blended Composite Optimization combining Stacking Sequence Tables and a Modified Shepard's Method

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1. Abstract
This article presents a computationally-efficient optimization tool for stacking sequence design of blended composite structures. In this tool, blended laminates are designed using stacking sequence tables (SST), coupled with a suitable genetic algorithm (GA). The SST approach guarantees complete-blending, ensuring manufacturability of the optimized design. The concept of successive structural approximations is implemented to improve computational efficiency. Optimizations are carried out on the approximations of responses rather than actual responses themselves, thus reducing the number of expensive design analyses. A recently-proposed modified Shepard's interpolation enriches the quality of the approximations used, by constructing multi-point approximations using the elite designs of the previous iterations. The generality and efficiency of the algorithm is further improved by directly approximating panel loads, thus enabling implementation of a wide range of stress-based design criteria.
An analytical multi-panel blended composite problem is presented as an application. The results show that completely blended and feasible stacking sequence designs can be obtained, having its structural performance close to the theoretical continuous optimum itself.
2. Keywords: stacking sequence optimization, blended composites, structural approximations.

3. Introduction
The use of composite materials in today's aerospace industry is experiencing a strongly increasing trend. The superior mechanical properties of composites and the ability to tailor their properties efficiently has been a major reason for this increased focus on its research and application. For practical design purposes, the ply angles and thickness of composites are usually restricted to a discrete set. The vast design space, coupled with the discrete nature of the design variables poses a tricky task of optimally designing composites.

The focus of this article is toward the design of efficient and manufacturable composite structures with varying stacking sequences in different regions. Spatially varying ply layups are necessary to efficiently tackle local load requirements. However, unless these individual zones or panels are designed correctly, abrupt ply-angle changes or ply-drops may occur, degrading the structural integrity of the component.

In order to overcome this, the concept of laminate blending [1] was introduced. Blending accounts for continuity of material and fibre content between adjacent panels having different stacking sequences. A blended design hence increases manufacturability and structural integrity. The optimization of blended composite designs has been well-studied and presented in [2–6].

Irisarri et al. [7] present a technique for achieving fully-blended designs by optimizing stacking sequence tables (SST) using a genetic algorithm (GA). In addition to guaranteeing fully-blended designs, optimizing using SST provides a detailed manufacturing insight of the ply-drop and transition region between adjacent panels. Furthermore, the GA takes into account several composite effects such as resin accumulation, free-edge delamination and transverse matrix cracking, by implementing industry-standard guidelines as part of the optimization.

In the design of composite structures on a practical-scale using GAs, an important challenge pertaining to computational costs arises from the large number of designs that need to be analysed. The concept of successive structural approximations [8] helps to reduce the computational costs by optimizing on approximations of the responses rather than on expensive responses themselves. Using response approximations in GAs to optimize for stacking sequence have been presented in [4, 9–12].

Irisarri et al. [13] present an effective approach in improving the quality of the approximations used. This is achieved by constructing multi-point structural approximations using a modified Shepard's method. As a result of the improved quality, the number of FE analyses required in the optimization is shown to be significantly reduced.

The stacking sequence design tool presented here uses the GA for SST [7] as the optimization algorithm. The GA was extended to account for load re-distribution using the optimization strategy presented in [13] for improved computational efficiency. In this work, panel loads are directly used as the responses to be approximated using which, structural responses like buckling and strain are obtained analytically. This novel approach provides the potential of including a wide range of stress-based responses in the optimization using in-house strength prediction tools.

Results from the weight-optimization of a multi-panel stiffened composite plate demonstrate the efficiency of the developed framework. Fully-blended designs having its performance reasonably close to the theoretical optimum were achieved, while requiring a low number of design FE analyses.

4. Optimization Framework

In a successive approximation technique as implemented here, optimizations are carried out on approximations of structural responses, followed by a design update with an FE analysis. This iterative optimization and update helps to reduce the number of required FE analyses.

The structural approximations used in this work are based on the generic formulation presented in [4] as

$$\tilde{f} = \sum_{i=1}^{n} (\Psi_i^m : A_i + \Psi_i^b : D_i + \Phi_i^m : A_i^{-1} + \Phi_i^b : D_i^{-1} + \alpha_i h_i) + c \tag{1}$$

where $\tilde{f}$ is the approximated response, A_i and D_i are the in-plane and bending stiffness matrices of the i^{th} design region or panel and n is the total number of design regions in the structure. The terms Ψ_i^m, Ψ_i^b, Φ_i^m, Φ_i^b are the sensitivities of the response with respect to the membrane, flexural stiffness matrices and their inverses and α_i is the sensitivity with respect to the laminate thickness h_i. The : operation is the matrix contraction or dot product and is defined as the trace of the product of two square matrices.

4.1 Stacking sequence tables

The optimizer uses a GA for SST [7] as the optimization algorithm. An SST is an intuitive method to represent and design a blended composite structure. For efficient use with a GA, the entire SST is encoded using just three chromosomes

1. SST_{lam}: stacking sequence of the thickest laminate in the SST.
2. SST_{ins}: order of insertion of the plies from the thinner laminate to subsequent thicker ones.
3. N_{str}: number of plies in each of the R panels or regions in the structure.

By optimizing the three chromosomes, an optimal stacking sequence distribution in the blended panels and a safe ply-drop distribution between them are simultaneously obtained.

The GA for SST was extended in the present work to handle multiple independent skins. The term skin hereby denotes a region of the structure locally blended within the panels in that region. For designs of a practical-scale like aircraft wings, the structure is usually manufactured in segments before being joined together. Ensuring complete blending over the entire structure is unnecessary and only restricts the design space. In a multiple-skin optimization, each skin is characterized by its own genotype, subjected independently to the various GA operators. Such a blended scheme is more appealing from an industrial perspective, while also enlarging the available design space.

4.2 Modified Shepard's method - evaluation of panel loads

The present tool uses load approximations to approximate only the panel loads. Vanderplaats et al. [14] present a similar approach in their optimization of isotropic plates. The motive behind such an approach is that once the panel loads in the structure are known, in-house analytical tools can be efficiently used to evaluate a multitude of structural responses, e.g., local buckling, strength at ply-level etc.

The approximated panel load in the k^{th} panel, constructed at a point i can be formulated similar to Eq. 1 as,

$$\tilde{N}_{i_k} = \sum_{j=1}^{n} (\Psi_{j,k}^m|_i : A_j + \Psi_{j,k}^b|_i : D_j + \Phi_{j,k}^m|_i : A_j^{-1} + \Phi_{j,k}^b|_i : D_j^{-1} + \alpha_{j,k}|_i h_j) + c_{i_k} \tag{2}$$

where $\Psi_{j,k}^m$, $\Psi_{j,k}^b$, $\Phi_{j,k}^m$, $\Phi_{j,k}^b$ and $\alpha_{j,k}$ are the sensitivities of the panel loads in the k^{th} panel to the respective laminate properties of the j^{th} panel and n is the total number of panels in the structure.

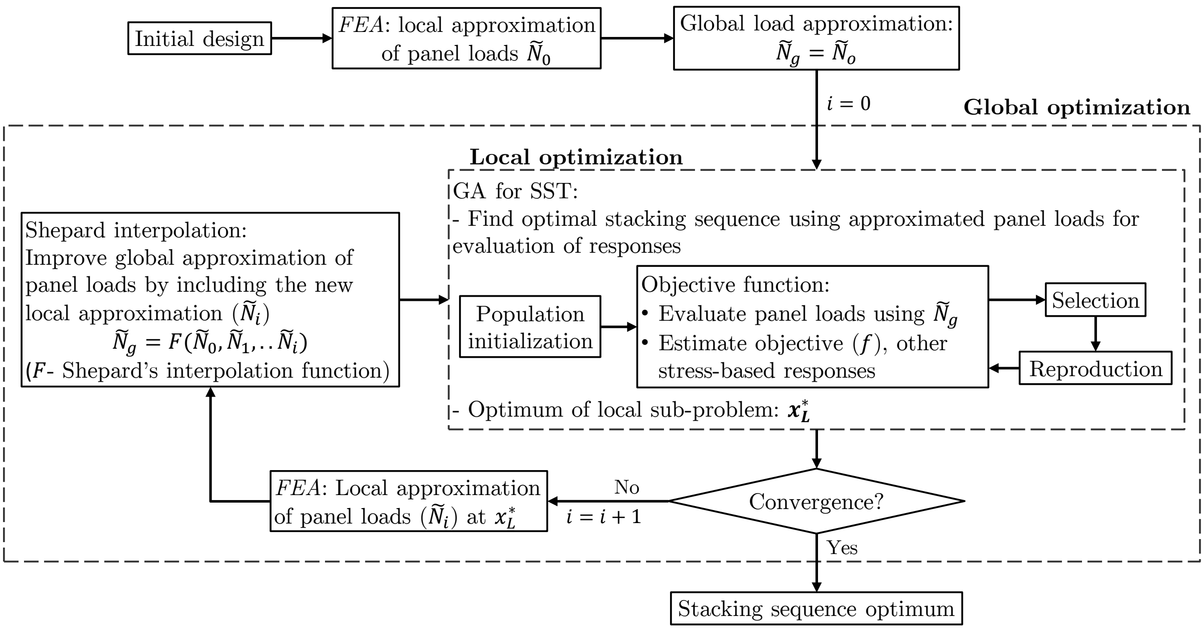

Figure 1: The optimization tool for the discrete optimization step - using GA for SST as the optimization method, Shepard's interpolation for approximation-improvement - assembled within a successive approximation framework.

The approximation in Eq. 2 is a single-point approximation and as such, is accurate in value and derivative only at the point i where it is constructed. The modified Shepard's method presented in [13] improves the quality of the approximation by constructing a multi-point global interpolant using information from several previous points. The actual panel loads $\tilde{N}$ are then evaluated using the multi-point Shepard's approximation constructed from all previous local approximations as

$$\tilde{N} = \frac{\sum_{i=1}^{n_i} w_i \tilde{N}_i}{\sum_{i=1}^{n_i} w_i} \tag{3}$$

where $\tilde{N}_i$ is the local approximation constructed at the i^{th} Shepard point (Eq. 2), n_i is the total number of previous design points and w_i is the interpolation weight [13].

In effect, the modified Shepard's interpolation ensures that as more points are added to the global approximation, the accuracy of the response surface increases over the entire spectrum of previous points leading to a much faster convergence.

4.3 Optimization process

The optimization tool thus combines two core constructs: a GA for SST and a modified Shepard's interpolation. The GA for SST utilizes a multi-point approximation for the evaluation of the panel loads. The GA itself is positioned within a successive approximation framework to account for load re-distribution, whereby new single-point local approximations, Eq. 2, are added after each subsequent global loop to the multi-point Shepard approximation, Eq. 3. The entire optimization procedure can hence be summarized from Fig. 1 as follows:

1. Perform FE analysis at a starting design to obtain first single-point approximation of the panel loads.

2. *Local optimization* - obtain the optimal stacking sequence design of the local problem using the approximated loads to evaluate the structural responses. The GA for SST is utilized for this step.

3. If the design has not converged, construct a new single-point approximation at the optimal design obtained from the sub-problem in Step 2, with an FE analysis. Convergence here occurs when there is no change in the objective of the optimal GA design obtained from two successive global loops.

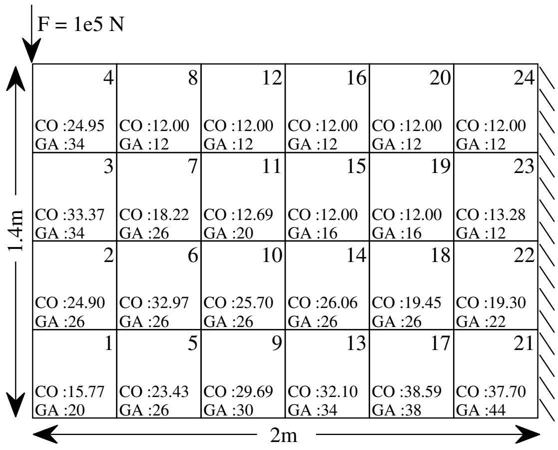

Figure 2: Problem definition and comparison of thickness distribution in the panels for the continuous optimum (CO) and GA optimum (GA).

4. Formulate the multi-point load approximation (Eq. 3) using previous single-point approximations (Eq. 2). Repeat Step 2.

A converged design in Step 3 corresponds then to the optimal stacking sequence design.

5. Results and discussions

A 4X6 multi-panel analytical problem (Fig. 2) was chosen to study the performance of the proposed tool. The objective of the optimization was weight-minimization of the blended composite structure, subjecting the individual panels to buckling, laminate strength and laminate robustness [15] constraints. The stacking sequence of the individual panels were optimized while the properties of the stiffeners were kept a constant; $E = E_1$, $A = 10^{-3}m^2$. The material properties of the Carbon/Epoxy listed in Table 1 with a ply thickness of $0.121mm$ were used. Ply orientations were restricted to steps of 15°. Additionally the following guidelines [7] from the SST were also enforced: laminate balance and symmetry, ply contiguity with a maximum of 4 contiguous plies, ply-disorientation and outer ±45 plies for damage tolerance.

A two dimensional membrane FE-routine was used to solve for the panel loads and their sensitivities. The separable approximations were constructed using only sensitivities with respect to the in-plane stiffness matrix Ψ_i^m. The local approximation from Eq. 2 is hence reduced to

$$\tilde{N}_{i_k} = \sum_{j=1}^{n} \Psi_{j,k}^m|_i : A_j + c_{i_k} \tag{4}$$

For the sake of comparison, the mass of the designs presented have been normalized with respect to the optimal mass obtained independently using a lamination-parameter based optimizer [16] - *11.351kg*. The design obtained from this continuous optimization can be conveniently termed the continuous optimum. As such, the continuous optimum represents the theoretical upper-bound in performance that can be achieved, since a lamination parameter-based continuous optimization assumes both ply-angles and thickness as continuous variables, while also not including the required constraint on blending. It is hence a convenient measure of comparison for the discrete designs obtained.

5.1 Optimal stacking sequence design

The optimal, fully-blended and feasible design obtained from the proposed tool was found to have a normalized mass of *1.097*, after just 14 global iterations. This increase in weight of $\sim 10\%$ over the continuous optimum is reasonable considering: the discrete stacking sequence design is fully blended, includes practical design guidelines and is limited by the discrete nature of the ply angles and thickness. The stacking sequence of this GA optimum can be obtained from its genotype and corresponding SST, presented in Table 3.

5.2 Optimization with multiple skins

Of particular interest to large-scale problems is the ability to design blended structures, where the blending

Table 1: Material properties: AS4/3501-6 [17]

E_{11} (GPa)	E_{22} (GPa)	G_{12} (GPa)	ν_{12}	ρ (kg/m^3)
142	10.3	7.2	0.27	1570
X_t (MPa)	X_c (MPa)	Y_t (MPa)	Y_c (MPa)	S (MPa)
2280	1440	57	228	71

Table 2: Mass optimization with multiple blended skins - normalized objective and critical constraint failure (the set of panels within each pair of braces constitute an independent skin)

Independently-blended skins	Objective	Critical constraint failure
1 - {1-24}	**1.097**	1.010
2 - {1-12}, {13-24}	**1.079**	1.004
2 - {1,2,5,6,9,10,13,14,17,18,21,22}, {3,4,7,8,11,12,15,16,19,20,23,24}	**1.056**	1.005
3 - {1-8}, {9-16}, {17-24}	**1.062**	1.001

is enforced in segments. The results in Table 2 show the optimal designs when the 24-panel structure is comprised of 1, 2 and 3 independently-blended skins.

As can be expected, an increase in the number of skins leads to a weakening on the requirement of blending, since blending is now enforced over a smaller number of panels. The resultant increase in design space leads to a reduction in mass as the number of skins increases. The term critical constraint failure here denotes the lowest factor of safety among all constraints. A value greater than 1 would hence imply a feasible design.

6. Conclusions

This article proposes an efficient optimization tool for blended composite design. The proposed approach combines two techniques: a GA using stacking sequence tables and multi-point approximations using a modified Shepard's interpolation method. A novel approach of directly approximating the structural loads is presented in this work. Working with panel loads directly is consistent with an industrial quick-sizing approach, providing the potential to include a large range of stress-based criteria in the optimization using in-house tools.

The performance of the tool is studied on a 24-panel composite blending problem. The results show that fully blended, feasible and guidelines-adhering stacking sequence designs can be obtained having its performance comparable to the theoretical optimum itself. Of equal importance, a low number of FE analyses required to reach the optimal design also show the computational efficiency of the proposed method.

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Table 3: Genotype and SST for the single-skin GA optimum. From the SST and the ply-distribution prescribed in chromosome N_{str}, the actual stacking sequence of each panel can be interpreted (only the symmetric half of the SST is shown here).

Chromosome	
N_{str}	[20 26 34 34 26 26 26 12 30 26 20 12 34 26 16 12 38 26 16 12 44 22 12 12]
SST_{lam}	[-45 -45 -30 -45 -15 30 60 60 15 45 90 45 45 60 90 -60 90 -60 -75 -60 90 75]
SST_{ins}	[0 4 6 2 0 7 10 12 0 3 5 1 0 15 14 16 0 11 9 13 0 8]
Optimum Solution 1.	
Mass = 12.456 kg. Min. buckling factor = 1.010. Min. strain factor = 3.794	

	N_{min}																N_{max}
no. of plies:	12		16		20		24		28		32		36		40		44
	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45
					-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45
							-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30
			-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45	-45
	-15	-15	-15	-15	-15	-15	-15	-15	-15	-15	-15	-15	-15	-15	-15	-15	-15
								30	30	30	30	30	30	30	30	30	30
											60	60	60	60	60	60	60
													60	60	60	60	60
	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
				45	45	45	45	45	45	45	45	45	45	45	45	45	45
						90	90	90	90	90	90	90	90	90	90	90	90
		45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
																60	60
															90	90	90
																	-60
	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
												-60	-60	-60	-60	-60	-60
										-75	-75	-75	-75	-75	-75	-75	-75
														-60	-60	-60	-60
	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90
									75	75	75	75	75	75	75	75	75

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