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Cross Sections and Prestressing Forces Optimizations of Prestressed Concrete Beams

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1. Abstract
In many cases prestressed concrete structures are more advantages than ordinary reinforced concrete structures. However, designing prestressed concrete structures that fulfil the optimum criteria is not an easy task. This is because there is an interaction between the size of cross sections and prestressing force that has to be given to the structures. Optimization of prestressed concrete structures becomes a challenging task for most structural designers. This paper considers optimization procedures for prestressed concrete beams. For obtaining prestressing force, the moment coefficient method, which is the ratio of moment due to prestressing force divided by prestressing force, is used in the computation. In statically determinate structures this coefficient exactly is the eccentricity between center of gravity of the section and the center of gravity of the prestressing steel. However, in statically indeterminate structures, this ratio is not the same as the eccentricity; due to the presence of the secondary moments. This leads to the concept of the moment coefficient-β. The optimization of both cross sections and prestressing force is carried out using real coded genetic algorithms (RCGAs), which has been successfully applied in many problems. One of the advantages of RCGAs is that it has the ability to explore the unknown domains that might be difficult to achieve by using binary coded. To show the effectiveness of the method, numerical examples are carried out using the proposed method. In the numerical examples, the optimization of cross sections and prestressing forces of both simple and continuous beams are considered. It can be shown that the method is effective to obtain the optimum cross section as well as prestressing force.
2. Keywords: prestressed concrete, optimization, real coded genetic algorithms, indeterminate prestressed structures.

3. Introduction

When the span of structures tends to be longer and longer, the needs of light structures becomes essential. Prestressed concrete is one of the solution to reduce the dimension of the members. With the simple concept, i.e., to reduce the tensile stress in concrete by applying the prestressing force, while maintaining the compressive force within the allowable stress, prestressed concrete has been receiving attention from designers. By using prestressed concrete it is possible to design longer structures compare to ordinary reinforced concrete. In another word, it is possible to achieve more efficient prestressed concrete members in contrast to ordinary reinforced concrete ones for the same loading. Other advantages of prestressed concrete are much less cracking, rapid construction and better quality control compared to ordinary reinforced concrete construction. Despite those advantages, designing prestressed concrete becomes more complex. This is due to the interaction between prestressing force and other requirements such as allowable stresses, which are also the function of the prestressing forces.

One method to design prestressed concrete members is by using the famous Lin's load balancing method [1], which is very simple, especially for one-span simple beams. For complex members, such as continuous beams, there will be a secondary moment due to the hyperstatic forces presence in statically indeterminate structures. In addition, for continuous beam the smooth transition of the cable profile should be maintained in the middle supports that render the application of the load balancing method. In order to alleviate this difficulty, the use of moment coefficient-β method has been proposed in [2-3]. This method has the ability also to avoid searching the concordant cable, which is usually very tedious. In statically determinate prestressed structures the coefficient-β turns to, exactly, the eccentricity of the tendon with respect to the center of gravity of concrete sections.

Recently optimization methods have also received a considerably attention from the structural designers [4-5]. These include size, shape and topology optimizations [6-7]. The algorithms used to optimize the problems include particle swarm optimizations, genetic algorithms, and ant colony methods [6-9]. In this paper real coded genetic algorithms (RCGAs) that have been used in control system [10] is used to optimize the cross sections and prestressing force of prestressed beams.

4. Design of Determinate and Indeterminate Prestressed Beams

The elastic design of prestressed concrete beams is to ensure that the stresses under service load conditions are within the allowable stresses as follows:

$$\sigma_L \le \sigma \le \sigma_U \qquad (1)$$

where σ_L = allowable compressive stress (lower limit), σ_U = allowable tensile stress (upper limit), and the stresses at the extreme fibers σ can be obtained as

$$\sigma = -\frac{F}{A_c} \pm \frac{M_F\, y_f}{I_c} \pm \frac{M\ \ y_f}{I_c} \qquad (2)$$

in which F = prestressing force, A_c = cross section area, M_F = moment due to prestress, y_f = distance from the extreme fibers to the center of gravity of the concrete section, I_c = second moment area of the section, and M = moment due to external loads.

The moment due to prestress M_F can be obtained as:

$$M_F = \begin{cases} F \times e \cdots\cdots\cdots \text{for statically determinate structures} \\ F \times \beta \cdots\cdots\cdots \text{for statically indeterminate structures} \end{cases} \qquad (3)$$

where e = the eccentricity of center of gravity of the steel (cgs) with respect to the center of gravity of the concrete section (cgc), and β = prestressing moment coefficient. Eq. (3) clearly indicates that for determinate structures e = β. The coefficient β can be obtained by dividing moment due to prestressing force (as the effect of the equivalent load from prestressing) by the magnitude of prestressing force in the tendons [2-3]. Throughout the paper, notation β will be used as a general case. Eq. (3) also means that here we have already considered the effect of the secondary moment due to the hyperstatic moment in the statically indeterminate prestressed concrete structures.

Following [2-3], at transfer (initial condition), the stress at the top fiber can be obtained as follows:

$$\sigma_{ci} \le -\frac{F_i}{A_c} - \frac{M_{Fi}\, y_t}{I_c} - \frac{M_{DL}\, y_t}{I_c} \le \sigma_{ti} \qquad (4)$$

where F_i = prestressing force at transfer, $M_{Fi} = F_i \times \beta$, y_t = neutral axis distance to top fiber, M_{DL} = moment due to dead load, σ_{ti} = allowable tensile stress in concrete at transfer, and σ_{ci} = allowable compressive stress in concrete at transfer.

Similarly, the stress at the bottom fiber has to follow:

$$\sigma_{ci} \le -\frac{F_i}{A_c} + \frac{M_{Fi}\, y_b}{I_c} + \frac{M_{DL}\, y_b}{I_c} \le \sigma_{ti} \qquad (5)$$

where y_b = neutral axis distance to top fiber

At the final condition, the stress at the top fiber has to follow:

$$\sigma_c \le -\frac{F}{A_c} - \frac{M_F\, y_t}{I_c} - \frac{M_{TL}\, y_t}{I_c} \le \sigma_t \qquad (6)$$

where F = prestressing force at the final stage after loss of prestress, $M_F = F \times \beta$, M_{TL} = moment due to the total load, σ_t = allowable tensile stress at the final condition, and σ_c = allowable compressive stress at the final condition.

Similarly, the stress at the bottom fiber has to follow:

$$\sigma_c \le -\frac{F}{A_c} + \frac{M_F\, y_b}{I_c} + \frac{M_{TL}\, y_b}{I_c} \le \sigma_t \qquad (7)$$

4.1. Prestressing Force Governed by Stress Conditions at Transfer

For simplicity, we take the notations:

$$r = \sqrt{\frac{I_c}{A_c}}, \qquad Z_t = \frac{I_c}{y_t}, \qquad Z_b = \frac{I_c}{y_b} \qquad (8)\text{-}(10)$$

At transfer, when the stress at the top fiber is in tension, and considering Eqs. (8) and (9), after several manipulations one can obtain[2-3]:

$$F_i\left(-\beta - \frac{r^2}{y_t}\right) < \sigma_{ti}\, Z_t + M_{DL} \qquad (11)$$

Eq. (11) becomes

$$F_{i\,max} = \frac{\sigma_{ti} Z_t + M_{DL}}{\left(-\beta - \frac{r^2}{y_t}\right)}, \quad \text{for} \left(-\beta - \frac{r^2}{y_t}\right) > 0 \tag{12a}$$

$$F_{i\,min} = \frac{\sigma_{ti} Z_t + M_{DL}}{\left(-\beta - \frac{r^2}{y_t}\right)}, \quad \text{for} \left(-\beta - \frac{r^2}{y_t}\right) < 0 \tag{12b}$$

Similarly, when the resulting stress at the top fiber is a compressive stress, we can obtain as follows:

$$F_{i\,min} = \frac{\sigma_{ci} Z_t + M_{DL}}{\left(-\beta - \frac{r^2}{y_t}\right)}, \quad \text{for} \left(-\beta - \frac{r^2}{y_t}\right) > 0 \tag{13a}$$

$$F_{i\,max} = \frac{\sigma_{ci} Z_t + M_{DL}}{\left(-\beta - \frac{r^2}{y_t}\right)}, \quad \text{for} \left(-\beta - \frac{r^2}{y_t}\right) < 0 \tag{13b}$$

Eqs. (12)-(13) are the results to ensure that the stress at the top fiber is within the allowable stress.
Similarly, in order to make sure that the stress at the bottom fiber at transfer in Eq. (7) is within the allowable stress, considering Eq. (10) one obtains as follows:
When the resulting stress is tensile stress:

$$F_{i\,max} = \frac{\sigma_{ti} Z_b - M_{DL}}{\left(\beta - \frac{r^2}{y_b}\right)}, \quad \text{for} \left(\beta - \frac{r^2}{y_b}\right) > 0 \tag{14a}$$

$$F_{i\,min} = \frac{\sigma_{ti} Z_b - M_{DL}}{\left(\beta - \frac{r^2}{y_b}\right)}, \quad \text{for} \left(\beta - \frac{r^2}{y_b}\right) < 0 \tag{14b}$$

When the resulting stress is compressive stress:

$$F_{i\,min} = \frac{\sigma_{ci} Z_b - M_{DL}}{\left(\beta - \frac{r^2}{y_b}\right)}, \quad \text{for} \left(\beta - \frac{r^2}{y_b}\right) > 0 \tag{15a}$$

$$F_{i\,max} = \frac{\sigma_{ci} Z_b - M_{DL}}{\left(\beta - \frac{r^2}{y_b}\right)}, \quad \text{for} \left(\beta - \frac{r^2}{y_b}\right) < 0 \tag{15b}$$

Eqs. (14)-(15) will ensure that the stress at the bottom fiber is within the allowable stress.

4.2. Prestressing Force Governed by Stress Conditions at the Final Stage

By using similar steps, the resulting equations from the final stage condition, after loss of prestress can be obtained by considering:

$$F = \alpha F_i \tag{16}$$

where α = effective prestress coefficient after loss of prestress.
Following [2-3], when the resulting stress at the top fiber is governed by the allowable tensile stress:

$$F_{i\,max} = \frac{\sigma_t Z_t + M_{TL}}{\alpha\left(-\beta - \frac{r^2}{y_t}\right)}, \quad \text{for} \left(-\beta - \frac{r^2}{y_t}\right) > 0 \tag{17a}$$

$$F_{i\,min} = \frac{\sigma_t Z_t + M_{TL}}{\alpha\left(-\beta - \frac{r^2}{y_t}\right)}, \quad \text{for} \left(-\beta - \frac{r^2}{y_t}\right) < 0 \tag{17b}$$

When the resulting stress at the top fiber is governed by the allowable compressive stress:

$$F_{i\,min} = \frac{\sigma_c Z_t + M_{TL}}{\alpha\left(-\beta - \frac{r^2}{y_t}\right)}, \qquad \text{for } \left(-\beta - \frac{r^2}{y_t}\right) > 0 \tag{18a}$$

$$F_{i\,max} = \frac{\sigma_c Z_t + M_{TL}}{\alpha\left(-\beta - \frac{r^2}{y_t}\right)}, \qquad \text{for } \left(-\beta - \frac{r^2}{y_t}\right) < 0 \tag{18b}$$

When the stress at the bottom fiber is in tension:

$$F_{i\,max} = \frac{\sigma_t Z_b - M_{TL}}{\alpha\left(\beta - \frac{r^2}{y_b}\right)}, \qquad \text{for } \left(\beta - \frac{r^2}{y_b}\right) > 0 \tag{19a}$$

$$F_{i\,min} = \frac{\sigma_t Z_b - M_{TL}}{\alpha\left(\beta - \frac{r^2}{y_b}\right)}, \qquad \text{for } \left(\beta - \frac{r^2}{y_b}\right) < 0 \tag{19b}$$

Similarly, when the stress in the bottom fiber is in compression one can obtain:

$$F_{i\,min} = \frac{\sigma_c Z_b - M_{TL}}{\alpha\left(\beta - \frac{r^2}{y_b}\right)}, \qquad \text{for } \left(\beta - \frac{r^2}{y_b}\right) > 0 \tag{20a}$$

$$F_{i\,max} = \frac{\sigma_c Z_b - M_{TL}}{\alpha\left(\beta - \frac{r^2}{y_b}\right)}, \qquad \text{for } \left(\beta - \frac{r^2}{y_b}\right) < 0 \tag{20b}$$

Prestressing forces for indeterminate prestressed concrete beams can be obtained from Eqs. (12)-(20). For determinate structures, the coefficient $\beta = e$.

5. Optimization of Prestressing Force and Cross Sections

In this paper real coded genetic algorithms (RCGAs) which have been successfully applied in the control systems [10] are used to optimize the cross sections as well as the magnitude of the prestressing force. RCGAs have the advantages that they can explore the unknown domain of design variables [10]. This is due to mutation and crossover operators used in the RCGAs. The steps of GAs used in RCGAs are also slightly modified, where after crossover and mutation, a portion of new individuals are inserted in order to increase the variability in the population. In addition, elitist strategy [11] is also used in this paper. Flow chart of RCGAs used in this paper is depicted in Fig.1. The objective function J is taken as the cost of the material given by

$$J = C_f \times \frac{1}{W} \tag{21}$$

where C_f is coefficient to scale the objective function and W is the total cost of the material obtained from

$$W = W_c \times C_c + W_s \times C_s \tag{22}$$

in which C_c = unit cost of concrete, W_c = weight of concrete, C_s = unit cost of steel, and W_s = weight of steel.

5.1. Numerical Example 1

The first example is a simple beam having span = 15 m, f'_c = 30 MPa, f'_{ci} = 25 MPa, unit weight of concrete = 24 kN/m^3, unit weight of steel = 7.85 kN/m^3, unit cost of concrete = 1,500,000.00 unit/m3, unit cost of steel = 40,000.00 unit/kg, dead load = 9.1 kN/m, and live load = 5 kN/m. The cable profile is parabolic with no eccentricity at the end of the member. The cgs of the tendon at midspan is taken minimum 0.12 m from the bottom fiber. The minimum width of beam is set up to 0.4 m. The allowable stress is taken as follows: allowable tensile stress at initial $\sigma_{ti} = 0.25\sqrt{f'_{ci}}$, allowable compressive stress at initial $\sigma_{ci} = -0.60 f'_{ci}$, allowable tensile stress at the final stage $\sigma_t = 0.50\sqrt{f'_c}$, and allowable compressive stress at the final stage $\sigma_c = -0.45 f'_c$.

RCGAs are used to optimize the structures, where maximum generation is taken = 500, probability of crossover =

0.8, probability of mutation = 0.1, portion of inserted new individuals = 10% of the total population, population size = 15. The scale factor C_f is taken $= 10^8$ and the effective prestress coefficient $\alpha = 0.8$.

After 500 generations the resulting beam section is b = 0.40 m, and h = 0.71 m. The prestressing force at transfer F_i is bounded between 1755.5 kN and 2693.6 kN. In this case the minimum force F_i = 1755.5 kN is used, resulting in the objective function J = 6.1518.

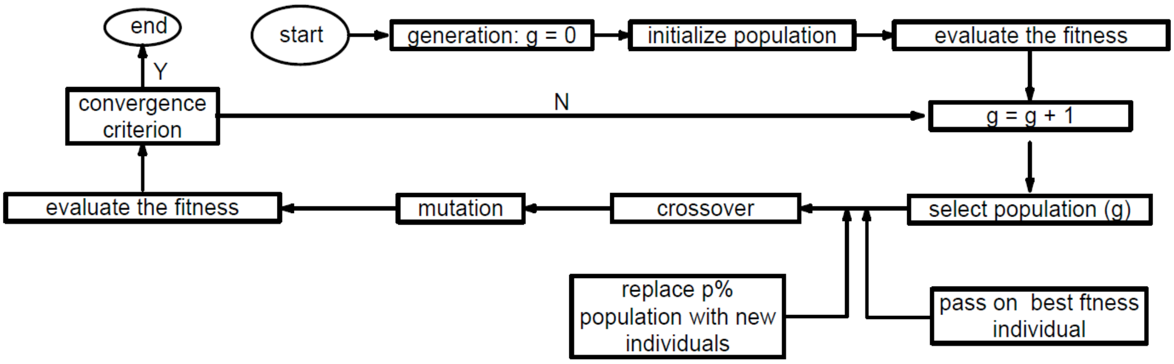

Fig.1. Flowchart for optimization

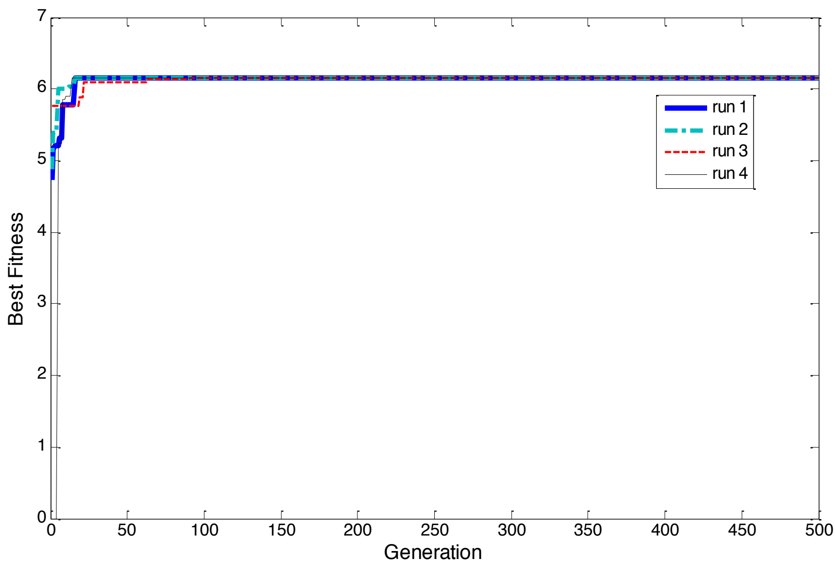

Fig. 2. Best fitness of example 1

5.2. Numerical Example 2

A three-span beam is designed as prestressed concrete member. Each span has 15 m in length. Other properties are the same as in numerical example 1. The same RCGAs are utilized, where four runs have been carried out to solve this problem. In this case C_f is taken $= 10^8$. The evolving best fitness for this case is shown in Fig. 3. After 500 generations the resulting design is b = 0.40 m, h = 1.28 m, the prestressing force at initial F_i = 553.7 kN to 685.1 kN. By using F_i = 553.7 kN, the resulting objective function J = 2.186.

6. Conclusions

The optimization method for obtaining the cross sections and prestressing forces has been discussed in this paper. Determination of the prestressing force is utilizing the coefficient-β so that it is not necessary to compute the secondary moment that presence in the statically indeterminate structures. The optimization method is utilizing real coded genetic algorithms. It is noted that for simple beams, the coefficient β is exactly equals to the eccentricity of the cable from the center of gravity of the sections. Two numerical examples have been carried out. The first is a one-span beam and the second is a three-span beam. It is demonstrated that the procedure is able to obtain the cross sections as well as the prestressing force that has to be given to the structures.

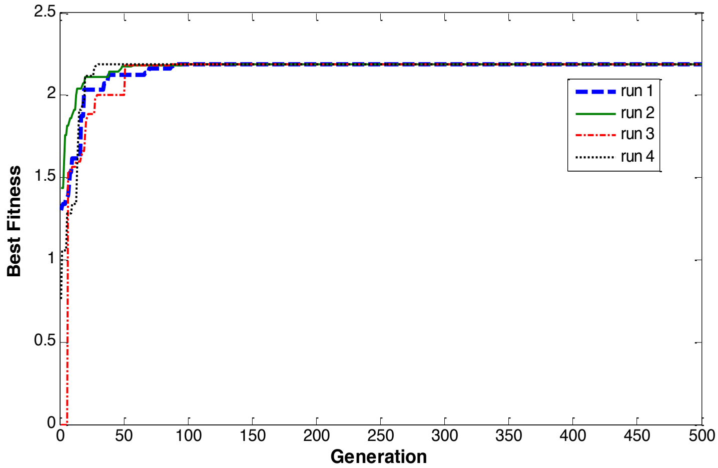

Fig. 3. Best fitness of example 2

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