

## Geometric Feature Identification from Topology Optimization Results

Guilian Yi<sup>1</sup>, Byeng D. Youn<sup>2</sup>, Nam H. Kim<sup>3</sup>

<sup>1</sup> Seoul National University, Seoul, Republic of Korea, flybirdyi@126.com

<sup>2</sup> Seoul National University, Seoul, Republic of Korea, bdyoun@snu.ac.kr

<sup>3</sup> University of Florida, Florida, US, nkim@ufl.edu

### 1. Abstract

Topology optimization yields an overall layout of a structure in the form of discrete density (e.g., SIMP) or continuous boundary geometry (e.g., level-set method). One of important drawbacks, however, is that it leads to a geometry with zigzag boundaries and/or irregular shapes, which is difficult to be interpreted for manufacturability, as well as to be utilized in subsequent applications such as shape optimization. It is considered the most significant bottleneck to interpret topology optimization results and to produce a parametric CAD model that can be used for shape optimization. The objective of this paper is to interpret geometric features out of a topology design to minimize human intervention in producing a parametric CAD model. The active contour method is first used to extract boundary segments from the greyscale image of topology optimization. Using the information of roundness and curvature of segments, simple geometric features, such as lines, arcs, circles, fillets, extrusion and sweep, are then identified. An optimization method is used to find parameters of these geometric features by minimizing errors between the boundary of geometric features and that of actual segments. Lastly, using the parametric CAD model, surrogate-based shape optimization is employed to determine the optimal shape. The entire process is automated with MATLAB and Python scripts in Abaqus, while manual intervention is needed only when defining geometric constraints and design parameters. 2-D beam and plate structures are presented to demonstrate effectiveness of the proposed methods.

**2. Keywords:** geometric features identification, active contour method, topology optimization, shape optimization, section optimization

### 3. Introduction

As one of the most active research topics in the field of structural optimization, topology optimization yields a prediction of the structural type and overall layout of materials. It can obtain robust results based on well-developed numerical approaches, and has received more and more research attentions recently because of its great potential of application to many industrial areas. Since most topology optimization approaches are element-based, where the initial design space is discretized by uniform rectangular finite elements and the design variables are assumed to be constant within each finite element, it is efficient in computation and has been applied successfully for solutions of many industrial optimization problems. However, one of important drawbacks of topology optimization is that, it is difficult to interpret topological results in terms of geometric features for conventional manufacturability. The optimal topological results often come out as greyscale images with zigzag boundaries and irregular shapes. They are not only difficult to make in the perspective of manufacturability, but also cannot be used directly in subsequent applications such as section and shape optimization. These problems are due to the lack of geometric boundary features and model parameterization. Many engineering applications require a shape of parameterized and smooth geometry, especially for manufacturing. There is a gap in automated geometric information extraction techniques in order to seamlessly integrate topology optimization with section and shape optimization. Therefore, there are needs to identify geometric features out of topology design and to obtain simple and smooth shapes for achieving cost-efficient design and for manufacturing components economically.

In the literature, some conventional methods were proposed to identify geometric boundaries by using image interpretation [1-2], density contour [3-4], geometric templates [5-6] and B-spline curves [7-8]. However, drawbacks of these methods still exist and make it difficult to achieve an efficient way for geometric features identification from topology results. For example, too many manual interventions are required in image processing, no parameterized model or geometric features can be obtained from density contours, limited geometric features can be represented by templates, and B-spline is expensive in the perspective of manufacturing. All of those shortcomings of existing geometric boundary detection methods drove us to interpret geometric features out of a topology design to minimize human intervention in producing a parametric CAD model.

In this paper, the geometric features identification techniques are limited to two dimensional structures, and simple geometric components are used for geometric features interpretation. Once the parameterized CAD model

for the topology result is obtained, shape optimization and section optimization are employed to fine tune the identified geometry and make sure the structural performances in line with that of the topology result. The whole procedures yield a structural design framework of integrated topology and shape/section optimization.

#### 4. Geometric features identification

It is important to identify the boundaries of geometric features from topology results. Although B-spline curves can provide nice shapes for a structure, and they are used commonly in daily life, it is very expensive, i.e. money and time, to manufacture them. On the other hand, more than 90% of machine parts are composed of simple geometric features, such as straight lines, fillets and circles. Therefore, straight lines, arcs, fillets, and circles are used as basic geometric features to interpret topology results and construct boundaries of geometry.

##### 4.1. Active contour method

Normally, element-based topology results are greyscale images, or result text file for element density values. This research can start with either of them. The image functions in Matlab, “imread( )” and “rgb2grey( )”, can read images and transform them into image data. Then, the image data are processed by the active contour method [9-10] to yield smooth and closed boundary segment data, which are composed of coordinate values. Each of the closed segments represents an outer/inner boundary.

A Matlab code based on active contour method made by Su [11] is used here to extract geometry boundaries from topology greyscale images. Figure 1(a) shows the initial greyscale image, and Figure 1(b) shows the extracted boundaries (red lines). Actually, the boundaries are expressed by a sequence of points, which are the blue crossed points as shown in Figure 1(c).

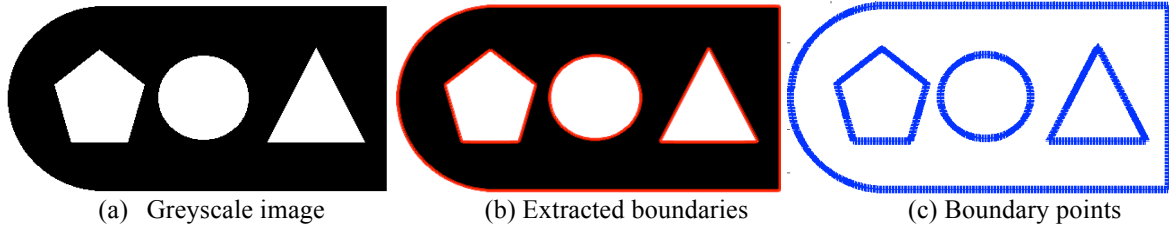


Figure 1: Image interpretation by using the active contour method

##### 4.2. Circles detection

Among many geometric features, circle is a very common shape, and different from straight lines, arcs and fillets. In order to identify a circle, “roundness” is used to detect circles from other geometric features. With identified boundary points of a closed loop, the centroid of a geometric feature is expressed as

$$(x_0, y_0) = \left( \frac{1}{n} \sum_{i=1}^n x_i, \frac{1}{n} \sum_{i=1}^n y_i \right) \quad (1)$$

where  $i$  is the number of boundary point, and  $n$  is the total number of the boundary points of the geometric feature.  $x_i$  and  $y_i$  are the coordinates of the  $i$ -th boundary point, and  $x_0$  and  $y_0$  are the coordinates for the centroid point.

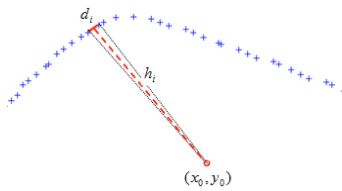


Figure 2: A portion of the boundary

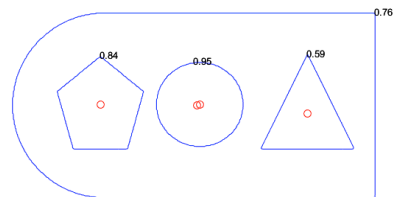


Figure 3: Roundness values

As shown in Figure 2, a small triangular is formed with two neighbored points and the centroid, whose base length and height can be expressed as

$$d_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2} \quad h_i = \frac{|k_i x_0 - y_0 + y_i - k_i x_i|}{\sqrt{1 + k_i^2}} \quad (2)$$

where  $k_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$ . Therefore, the perimeter and area of this geometric feature can be obtained as

$$P = \sum_{i=1}^{n-1} d_i \quad A = \sum_{i=1}^{n-1} \frac{1}{2} d_i h_i \quad (3)$$

where  $P$  and  $A$  denote the perimeter and area of a closed geometric feature, respectively.

The roundness of this geometric feature can then be defined by

$$m = 4\pi AP^{-2} \quad (4)$$

where  $m$  represents the roundness value of a closed geometric feature. A roundness value closer to 1.0 indicates that the object is approximately circle. Figure 3 shows the centroids (red circle points) and roundness values for the four geometric features. By setting the threshold of 0.9, circles can be identified first.

#### 4.3. Straight lines and fillets identification

After identifying circles, it is assumed that other closed-loop boundaries are composed of straight lines and fillets between two lines. Since the boundaries are identified and described by points, it makes sense to interpret a straight boundary section as a line segment according to its curvature. Starting from the initial point, slopes of lines connecting following points with the initial point are calculated. If these slopes change beyond a threshold, it is considered that the current line ends and a new line starts from the ending point. A fillet tangent to the two straight lines is used to express corner boundaries.

Figure 4 shows the geometric relationship of a fillet and two straight lines (red lines) on a portion of a geometric feature (blue cross points), and the relationship is formulated in Eq. (5) to calculate fillet center coordinates and tangent points for a given fillet radius. In Figure 4,  $n_1$  and  $n_2$  are, respectively, the numbers of two tangent points' locations in a closed geometric feature. The points between the  $n_1$ -th and  $n_2$ -th points are used to approximate a fillet tangent to two straight lines.

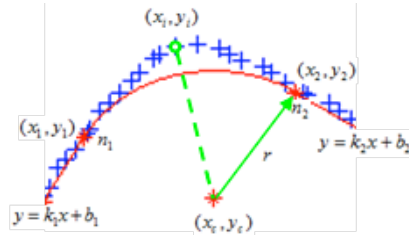


Figure 4: Geometric relationship of a fillet and a straight line

$$\begin{cases} \sqrt{(x_1 - x_c)^2 + (y_1 - y_c)^2} - r = 0 \\ \sqrt{(x_2 - x_c)^2 + (y_2 - y_c)^2} - r = 0 \\ (x_1 - x_c) + k_1(y_1 - y_c) = 0 \\ (x_2 - x_c) + k_2(y_2 - y_c) = 0 \\ k_1x_1 + b_1 - y_1 = 0 \\ k_2x_2 + b_2 - y_2 = 0 \end{cases} \quad (5)$$

where  $(x_c, y_c)$  denotes coordinates of the fillet center point, and  $r$  is the fillet radius.  $(x_1, y_1)$  and  $(x_2, y_2)$  are two tangent points at both ends of the fillet.  $k_1$  and  $k_2$  are the slopes, and  $b_1$  and  $b_2$  are the y-intercept of the two lines.

In order to estimate more accurate fillets, Eq. (6) is formulated to minimize the maximum distance from the initial boundary segment data points to the fillet.

Find  $r$

$$\text{Min } f(r) = \max_{i \in \{n_1, n_2\}} \left| \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} - r \right| \quad (6)$$

s.t.  $r > 0$

Once the optimal fillet radius is obtained from Eq. (6), the fillet center point and two tangent points can be determined by solving Eq. (5). Figure 5 shows a pentagon identified with 5 line segments and 5 fillets before and after fillet estimation. The blue lines are the initial boundaries, which are not smooth, and the red lines are the identified boundaries. The red star points on the red line are tangent points, and red star points inside of the boundaries are fillet center points. Arc can be detected in a same way as fillets.

The estimation process for fillets obviously alleviates the errors between final parameterized smooth shapes and initial topological shapes, and makes sure the shapes to be consistent with the geometric features. At last, the geometries will be output in forms of parameterized CAD model with straight lines, fillet, arcs and circles, and be

rebuilt in Abaqus.

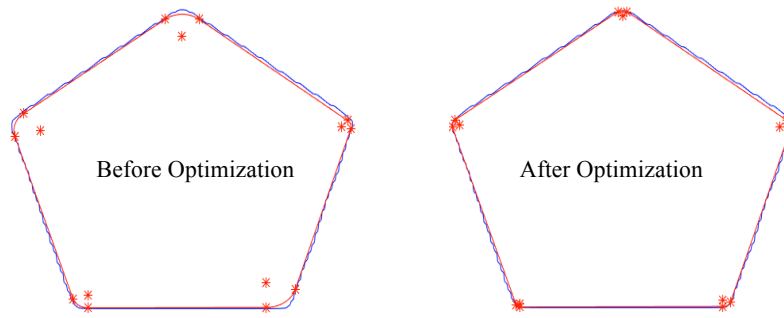


Figure 5: Before and after optimization for fillets

### 5. Section and shape optimization for identified structure

Although the boundary shapes are extracted from topology optimization results, the extracted geometry may not be considered an optimum geometry because the structural response can slightly be different from that of topology design. Therefore, a follow-up section or shape optimization is performed to fine tune the structural geometry for desired performances. The same optimization formulation used for topology optimization is better to be used for the section or shape optimization. During the follow-up optimization, no new geometry features will be introduced, but the initial features will be modified to find the optimum design.

In the follow-up optimization, there are a couple of things to be noted. Firstly, dimensions of the CAD model should be fully constrained so that the entire geometry can be regenerated after its parameters are changed. Secondly, since not all of the parameters are necessary for optimization, a limited number of dimensions can be selected as design variables, and their lower/upper bounds needs to be determined. The lower/upper bounds are selected such that the geometry can be well defined within these bounds. Since the selection totally depends on the experiences of designers, this parameterization process has to be done manually, and it is the only time that requires human intervention in the whole design procedure.

As the dimensions change, the structural shape and mesh change too. It causes numerical errors when we use finite difference method to calculate the sensitivities in optimization. In order to address the issue of mesh-related numerical errors, the surrogate model-based optimization approach is employed to perform the shape optimization. The optimization problems will be solved by the “fmincon” function in Matlab. The optimization for the CAD model is implemented automatically by using Matlab to execute Python scripts for Abaqus.

### 6. Numerical examples

In order to demonstrate the validity and capability of the proposed framework for geometric features identification, two numerical examples are tested here. The first example is topology result of a cantilever beam from Yi and Sui [12]. Section optimization is considered with beam elements. The second example is topology result of a clamped plate from Yi [13], and shape optimization is considered for the topology shape. The topology optimization of both structures was performed to minimize structural mass with a displacement constraint.

**Example 1: Beam element identification and section optimization.** It was required that the displacement at the loading point was no larger than 0.35mm. Figure 6 shows the grayscale image of the optimal topology result, where the minimum mass was 1990.17kg, and the displacement at the loading point was an active constraint.



Figure 6: Topology shape

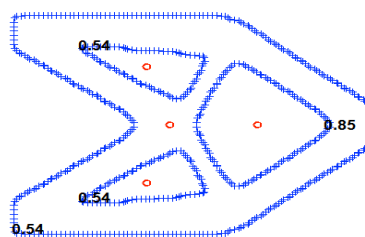


Figure 7: Boundaries and roundness

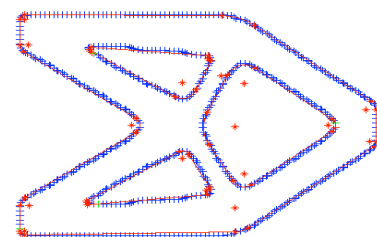


Figure 8: Straight lines and fillets

Figure 7 gives the boundary points and roundness values of closed boundary segments. By implementing the geometric features identification techniques on those boundary segments, the parameterized CAD model with straight lines and fillets is overlapped on top of the initial boundaries, which is shown in Figure 8. Although the fillets are determined by optimization, they are not used in section optimization, but used to determine the

approximations of straight lines. The middle lines of two closely parallel straight lines are determined one by one. They are crossed to each other and construct a beam element structure. As shown in Figure 9, the red lines are identified boundary line segments, and the green lines are beam elements of the cantilever beam.

Since the cantilever beam is symmetric, four section areas are selected as design objectives (shown in Figure 10). A rectangular profile is defined for each of them. Since the thickness of the beam is 1.0mm, the design variables are actually the heights of the rectangular sections of Sec1, Sec2, Sec3 and Sec4 separately. A lower bound of  $lb = [0.1, 0.1, 0.1, 0.1]$  (mm) and an upper bound of  $ub = [10, 15, 10, 15]$  (mm) are given for the design variables. Since the mesh on beam elements does not change during optimization, no surrogate model is required, and the section optimization is conducted directly. The optimum point is found at  $\mathbf{x}^* = [6.12, 7.81, 3.71, 6.71]$  (mm). The structural mass is 1981.13kg, and the displacement at the loading point is 0.35mm. Displacement contour of the beam with optimal sections is shown in Figure 11.

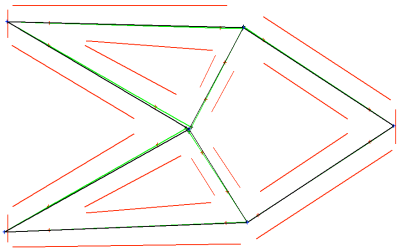


Figure 9: Beam element structure

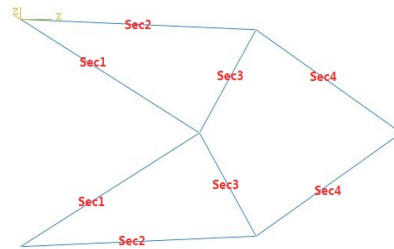


Figure 10: Design objectives

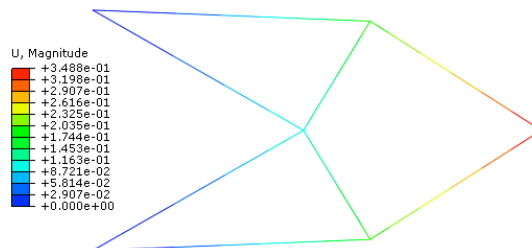


Figure 11: Optimal sections

**Example 2:** Plate shape identification and shape optimization. The displacement at the middle of the plate along the loading direction was required to be no larger than 0.74m. The result text file of the optimal density values is used for image interpretation. Figure 12 shows the topology shape (in black and grey) read by Matlab and boundaries (in red) detected by the active contour method. The total mass was 5.31kg, and the displacement at the middle was 0.74m. The exact boundaries are identified and shown in Figure 13, where the fillets and arcs are already optimized. The CAD model is rebuilt in Abaqus and the fully constrained geometry is shown in Figure 14.

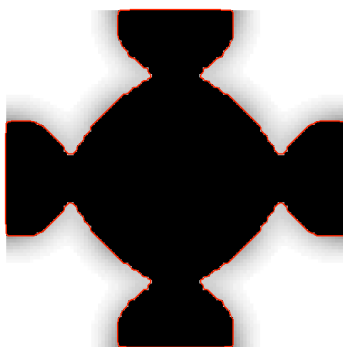


Figure 12: Topology shape

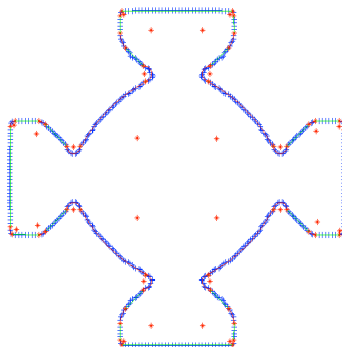


Figure 13: Identified boundaries

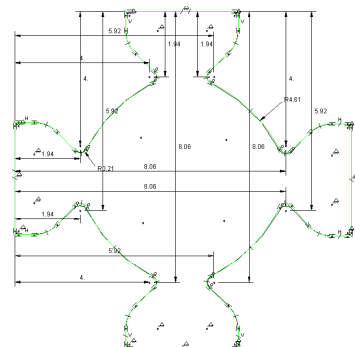


Figure 14: Fully constrained CAD model

Since the geometry is fully symmetric and the dimensions are related to each other, the dimensions for the short fillet, x-coordinate and y-coordinate of the fillet center point, and fillet radius, respectively, are considered as design variables  $x_1$ ,  $x_2$  and  $x_3$ , which are marked as pink lines in Figure 15. The shape optimization with a displacement constraint is performed on the identified geometry. The Kriging surrogate model is utilized to approximate the

relationship between three design variables and the displacement at the middle point of the plate. The optimal design is found at  $\mathbf{x}^*=[2.01, 4.14, 0.1]$  (m), where the structural mass is 5.00kg, which is reduced by 5.84% compared to the topology result, and the displacement at the middle point is 0.74m. Figure 16 gives a full dimensions of the optimal shape of the identified geometry. The displacement contour of the optimal shape is shown in Figure 17.

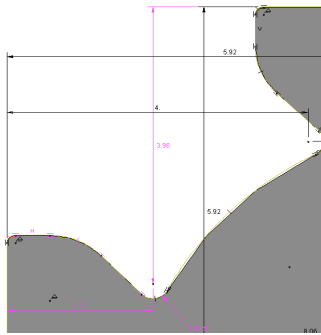


Figure 15: Design variables

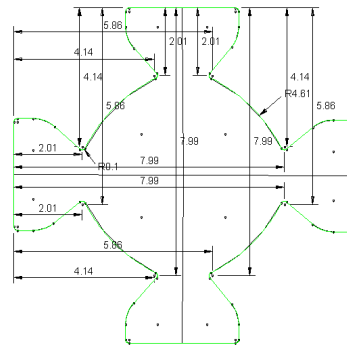


Figure 16: Optimal shape

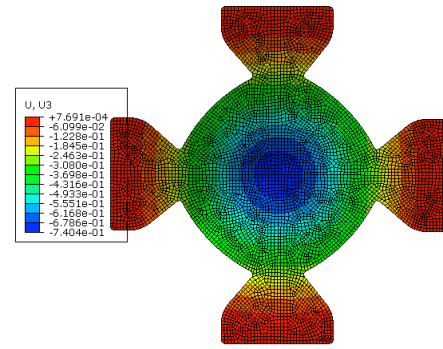


Figure 17: Displacement contour

## 7. Conclusions

In this paper, 2D geometry features are identified from topology optimization results, followed by section or shape optimization. Both numerical examples indicate that the proposed techniques for geometric features identification are valid and capable to interpret topology results. In addition, the proposed structure design framework shows that topology and shape/section optimization can be integrated to effectively obtain structural designs for manufacturing. However, some future work needs to be considered, for example, parameterizing boundary conditions to make process fully automatic, and applying this integration framework into 3 dimensional structures.

## 8. Acknowledgements

We gratefully acknowledge the financial assistance from the China Scholarship Council and the Brain Korea 21 Plus project in 2015.

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