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Reduced super beam based approach to finite element model updating of beam-type structures

Wensheng Wang[1], Haojie Wei[1], Zhonghua Hou[1]

[1] Department of Engineering Mechanics, Henan University of Science and Technology, Luoyang, China,
Email: wswang@live.cn

1. Abstract

A reduced super beam based finite element model updating technique for beam-type structures is proposed in this paper. The model reduction method is adopted to condense the entire beam-type structural model into a reduced super beam model with much less degree of freedom. And the eigensolutions and eigensensitivities are re-analyzed from the reduced eigenequation of the reduced super beam in the updating process, thus reducing the computational load of the traditional model updating methods which perform on the original structure. The modal dynamic property difference approach is adopted for updating the reduced super beam model and standard optimization techniques are used to find the optimal values of the structural parameters that minimize the difference. The effectiveness and efficiency of the proposed method are illustrated through a complicated stiffened cylindrical shell structure.

2. Keywords: Reduced super beam, model updating, beam-type structures, optimization

3. Introduction

Great progresses have been achieved in finite element method (FEM) modeling during the past decades. However, due to the uncertainties in the geometry, material properties and boundary conditions of the FEM model, the dynamic responses of a structure predicted by a highly idealized numerical model usually differ from the experimental results. For example, He et al. [1] reported that the differences between the experimental and numerical modal frequencies of an aircraft wing exceeded 10% for most modes and even reached 70% in some cases. Similarly, more than 60% difference was found between the analytical and measured frequencies of an aero-engine casing by Ma et al. [2]. Therefore, an effective model updating method is necessary to obtain a more accurate FEM model that are required in a large number of applications, such as optimization design, damage identification, structural control and so on[3].

In the past years, various FEM model updating methods have been developed and practically applied, which can be classified into two categories: one-step methods and iterative methods [4]. The former directly reconstruct the stiffness and mass matrices of the analytical model, and the symmetry, positive-definiteness and sparseness in the updated matrices cannot be preserved. The latter modify the physical parameters of the FEM model repeatedly to minimize the modal properties discrepancy between the analytical model and the measurement counterparts, which are becoming more popular. Optimization techniques are employed in most iterative model updating methods, the eigensolutions and sensitivity matrices of the analytical model must be calculated in each iteration [5]. As the analytical model of a practical structure in engineering usually comprises a large number of degrees of freedom (DOFs), it is very time-consuming to extract the eigensolutions and eigensensitivities from the large-size system matrices, especially for many uncertain parameters that need to be updated.

To address the computational difficulty, reduced model-based FEM model updating methods have been investigated. The substructure based model updating method has been studied [6-8], which is advantageous mainly in two aspects. Firstly, it is much easier and quicker to analyze the small system matrices for eigensolutions and eigensensitivities, as the original structure is replaced by smaller substructures. Secondly, the separated substructures are analyzed independently when applied to model updating. When the updating parameters are localized within parts of a structure, only one or more substructures containing the parameters are re-analyzed during model updating, and the other substructures are untouched. However, the construction of the reduced base needs a lot of intricate matrices calculation and the accuracy of the substructure based model updating method relies on the optimum selection of master modes in the substructures.

In this work, a reduced super beam based updating method for beam-type structure is presented. The reduced super beam method [9-10]，based on the plane cross section assumption and displacement interpolation function of beam，is a new model reduction method for beam-type structure. This paper intends to develop the reduced super beam method and apply it to calculate the eigensolutions and eigensensitivities for the sensitivity based model updating process. The modal dynamic property (frequencies and mode shapes) difference approach is adopted for updating the reduced super beam model. In particular, Eigensensitivities with respect to an updating parameter of the global structure is calculated from the derivative matrices of the reduced super beam, which can save a large amount of computational effort in the model updating process. A complicated stiffened cylindrical shell structure is employed to demonstrate the effectiveness and efficiency of the proposed method.

4. Reduced super beam method

The reduced super beam method is briefly introduced in this section for model updating purpose. Considering a complicated free-free beam-type structure shown schematically in Figure 1(left figure), Oxyz is the global coordinate system, in which the Ox axis is along the structural axis and Oy and Oz are in the cross section perpendicular to the structural axis.

As the first step of the new model reduction method, the structure is divided into several parts by a number of cross section which is perpendicular to the structural axis. The intersections of the cross sections with the structural axis are defined as the master nodes. The original FEM model, which may have hundreds of thousands DOFs, will be reduced to a super beam model with the master nodes. Each master node has six degrees of freedom, i.e., three translational and three rotational degrees of freedom.

To construct such a super beam, each structural part is modeled as a super beam element. That is, the nodal displacement field in each part is approximated by the generalized displacement of the two master nodes at its end through twice transformation (i.e., the first transformation is performed between the structural node and the projective node, while the second transformation is performed between the projective node and the master node as shown in Figure 1). The detail of the reduction method is as follows.

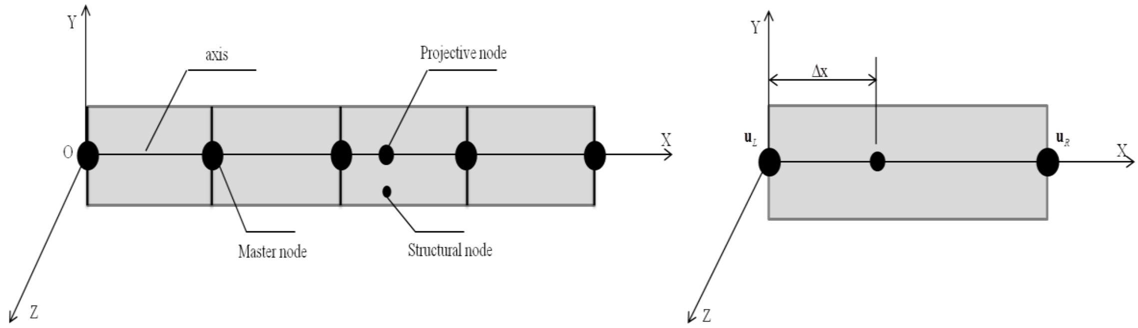


Figure 1: Principle of the model reduction method

In each part i, considering the deformation characteristics of beam-type structure, the well known plane cross-section assumption is applied to project the nodal displacements $\boldsymbol{u}_j$ of each structural node j to the rigid body motion vector $\boldsymbol{q}_{ji}$ of its corresponding projective nodes on the Ox axis. The relationship can be expressed as

$$\boldsymbol{u}_j = \boldsymbol{R}_j \boldsymbol{q}_{ji} \tag{1}$$

The expand form of Eq. (1) is

$$\begin{Bmatrix} u_{jx} \\ u_{jy} \\ u_{jz} \\ u_{j\theta x} \\ u_{j\theta y} \\ u_{j\theta z} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & z_{cj} & -y_{cj} \\ 0 & 1 & 0 & -z_{cj} & 0 & 0 \\ 0 & 0 & 1 & y_{cj} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} q_{jix} \\ q_{jiy} \\ q_{jiz} \\ q_{ji\theta x} \\ q_{ji\theta y} \\ q_{ji\theta z} \end{Bmatrix} \tag{2}$$

The six column of $\boldsymbol{R}_j$ is denoted by $\boldsymbol{R}_{j1}, \boldsymbol{R}_{j2}, \boldsymbol{R}_{j3}, \boldsymbol{R}_{j4}, \boldsymbol{R}_{j5}, \boldsymbol{R}_{j6}$ .The displacement vector $\boldsymbol{q}_{ji}$ of the projective node between the two master nodes of each part i can be obtained through displacement interpolation function of beam. Suppose the displacement of the master nodes is $\boldsymbol{u}_L = (u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1})^T$ (left) and $\boldsymbol{u}_R = (u_2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2})^T$ (right) respectively, as shown in figure 1(right figure). According to the finite element theory of frame structure, displacement transformation between the projective node and the two master nodes can be approximated by the interpolation function of beam element as

$$\boldsymbol{q}_{ji} = \boldsymbol{N} \left\{ \boldsymbol{u}_L, \boldsymbol{u}_R \right\}^T \tag{3}$$

The expand form of interpolation functions $\boldsymbol{N}$ can be found in paper [10]. By Eq. (1) and (3), the transformation relationship matrices between the displacement of each structural node j and the two master nodes in each part i can be written as

$$\overline{\boldsymbol{R}}_j = \boldsymbol{R}_j \boldsymbol{N} \tag{4}$$

The displacement of $\tilde{m}$ nodes in each part i are all projected to the two assumed master nodes of the part i. Using the transformation (4), the transformation equation of the part i can be defined as:

$$\boldsymbol{U}_i = \overline{\boldsymbol{T}}_i \left\{ \boldsymbol{u}_L , \boldsymbol{u}_R \right\}^T \tag{5}$$

where $\boldsymbol{U}_i = (\boldsymbol{u}_1, ..., \boldsymbol{u}_j, ..., \boldsymbol{u}_{\tilde{m}})^T$ is the nodal displacement vector of the $\tilde{m}$ FEM nodes in part i, and the transformation matrices between the $\tilde{m}$ FEM nodes and the two master nodes of the part i is $\overline{\boldsymbol{T}}_i = (\overline{\boldsymbol{R}}_1, ..., \overline{\boldsymbol{R}}_j, ..., \overline{\boldsymbol{R}}_{\tilde{m}})^T$, with size of $6\tilde{m} \times 12$. Each column in the transformation matrices $\overline{\boldsymbol{T}}_i$, actually a reduced base vector with explicit and localized form, can be obtained with very low computational cost. With this transformation(5), the mass and stiffness matrices of the super beam element are defined as

$$\boldsymbol{m}_s = \overline{\boldsymbol{T}}_i^T \boldsymbol{m} \overline{\boldsymbol{T}}_i , \boldsymbol{k}_s = \overline{\boldsymbol{T}}_i^T \boldsymbol{k} \overline{\boldsymbol{T}}_i \tag{6}$$

where $\boldsymbol{m}_s$ and $\boldsymbol{k}_s$ has dimension 12×12 .Once the super beam element is constructed by Eq.(6), it could be assembled to obtain a free-free reduced super beam model. For a uniform beam type structure, it is only necessary to construct the super element once, otherwise, it is needed to construct several super beam elements for non-uniform general beam type structure. In this reduction process, the retained DOFs in the reduced model preserve its physical characteristics and provide the possibility for further necessary manipulation. The accuracy of the reduced super beam model is further improved by modifying the stiffness matrices of super beam element and considering the effect of shear deformation in a rational way, and the interested reader can refer to paper [10].

Suppose the whole beam-type structure with s nodes ($6s$ DOFs) was divided into $p(p \ll s)$ parts, in other words, there are $p+1$ master nodes in all. A final reduced super beam model has DOFs ($6(p+1)$), which is much less than $6s$. The mass and stiffness matrices of the reduced super beam are defined as

$$\boldsymbol{K}_R = \sum_1^p \boldsymbol{k}_s \qquad \boldsymbol{M}_R = \sum_1^p \boldsymbol{m}_s \tag{7}$$

5. Model updating using modal property

Sensitivity-based FEM model updating method is the most frequently used updating method, and the general objective function combining the modal properties (frequencies and mode shapes) is usually represented as [6]

$$J(r) = \sum_i w_{\lambda i}^2 \left[\lambda_i \left(r \right)^A - \lambda_i^E \right]^2 + \sum_i w_{\phi i}^2 \sum_j \left[\phi_{ji} \left(r \right)^A - \phi_{ji}^E \right]^2 \tag{8}$$

where λ_i^E represents the eigenvalue corresponding to the ith experimental frequency, and ϕ_{ji}^E is the ith experimental mode shape at the jth experimental point. λ_i^A and ϕ_{ji}^A denote the corresponding eigenvalue and mode shape from the FE model, expressing as the function of the updating parameters r. $w_{\lambda i}$ and $w_{\phi i}$ are the weight coefficients due to the different measurement accuracy of the frequencies and mode shapes. The objective function is minimized by continuously adjusting the parameters r of the initial FE model through optimization process.

5.1 Sensitivity analysis

To find the optimal searching direction, sensitivity analysis is usually conducted to compute the rate of the change of a particular response quantity with respect to the change in a physical parameter. For the objective function, a truncated Taylor series of $J(r)$ is defined as

$$Z(r) = J(r) + [\nabla J(r)]^T (\Delta r) + \frac{1}{2} (\Delta r)^T [\nabla^2 J(r)]^T (\Delta r) \tag{9}$$

where Δr denotes a step vector from the current r. $\nabla J(r)$ and $\nabla^2 J(r)$ are the gradient and the Hessian of $J(r)$, respectively. After an iterative process, the optimized r^* is reached with $\nabla J(r) \approx 0$. The gradient and Hessian of $J(r)$ can be expressed by the sensitivity matrices as

$$\nabla J(r) = [S(r)]^T \left\{ 2 f(r) \right\} \qquad \nabla^2 J(r) = S(r)^T S(r) \tag{10}$$

where $f(r)$ encloses the weighted residuals $w_\lambda (\lambda(r)^A - \lambda^E)$ and $w_\phi (\phi(r)^A - \phi^E)$. The sensitivity matrices of the eigenvalues and mode shapes with respect to a parameter r can be expressed as

$$S_{\lambda}(r) = \frac{\partial \lambda(r)}{\partial r} \qquad S_{\phi}(r) = \frac{\partial \phi(r)}{\partial r} \tag{11}$$

The sensitivity matrices $S(r)$ may be determined analytically or by using the finite difference method [11].

5.2 Eigensolutions with reduced super beam method

The eigensolutions and eigensensitivity matrices can be calculated based on the classical eigenequation

$$\mathbf{K}\phi_i = \lambda_i \mathbf{M}\phi_i \tag{12}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices, λ_i and ϕ_i are the ith eigenvalue and eigenvector, respectively. In traditional model updating methods, the eigensolutions and eigensensitivities analysis based on the large-size system matrices is expensive, updating the FEM model of a large-scale structure usually involves a heavy workload and many runs are usually required to achieve the convergence of the optimization.

In the present paper, a beam-type structure is condensed to a reduced super beam based on the aforementioned reduced method, aiming at reducing the sizes of the stiffness and mass matrices and eliminating the expensive reanalyses of eigenproblems due to the variations of the updating parameters. The reduced solutions are then transformed to obtain eigensolutions and eigensensitivity of the global structure by using transformation matrix, which is an assemblage of $\overline{\boldsymbol{T}}_i$.

We also suppose that the stiffness and mass matrices depend linearly on the model parameter $\mathbf{r}$, which is often encountered in practical applications of model updating. Specifically, it is assumed that the mass and stiffness matrices of the original model take the form as

$$\mathbf{K}(\mathbf{r}) = \mathbf{K}_0 + \sum_{i}^{N_r} \mathbf{K}_i r_i \quad \mathbf{M}(\mathbf{r}) = \mathbf{M}_0 + \sum_{i}^{N_r} \mathbf{M}_i r_i \tag{13}$$

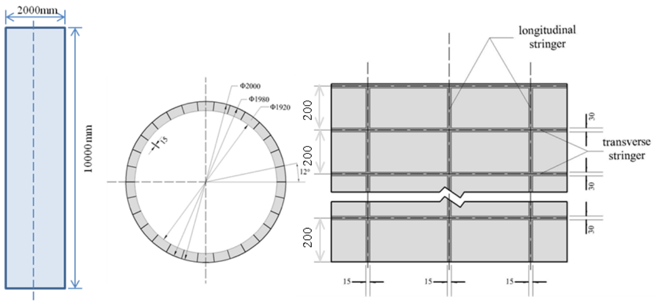
where $\mathbf{K}_0$, $\mathbf{M}_0$, $\mathbf{K}_i$ and $\mathbf{M}_i$ are constant matrices, independent of $\mathbf{r}$, and N_r is the number of structural model parameters to be updated. Then the construction of the super beam element is guided by the linear dependence so that the stiffness and mass matrix for each beam element depend linearly on only one of the parameters to be updated. The mass and stiffness matrices at the reduced level admit a similar representation as (14),

$$\mathbf{K}_R(\mathbf{r}) = \mathbf{K}_{R0} + \sum_{i}^{N_r} \mathbf{K}_{Ri} r_i \quad \mathbf{M}_R(\mathbf{r}) = \mathbf{M}_{R0} + \sum_{i}^{N_r} \mathbf{M}_{Ri} r_i \tag{14}$$

In order to save computational time, the constant matrices are computed and assembled once and, therefore, there is no need for this computation to be repeated during the iterations in optimization for model updating.

6. Numerical example: a stiffened cylindrical shell structure

To illustrate the feasibility and computational efficiency of the proposed method, a uniform typical stiffened cylindrical shell structure in free-free boundary condition is employed here as shown in figure 2. Figure 3 gives its FEM model with 4530elements, 3060 nodes and 18360 DOFs in total. The material of the structure is isotropic with Young's modulus $E = 7.0 \times 10^4 MPa$, Poisson's ratio $\nu = 0.3$, and mass density $\rho = 2.7 \times 10^{-3} g / mm^3$.



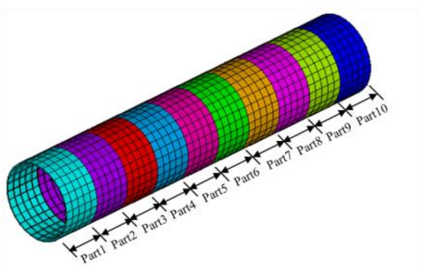


Figure 2: Typical stiffened cylindrical shell structure

Figure 3: FEM model of the cylinder structure

The original full FEM model was reduce to a super beam model of 50 uniform super beam elements in this paper, and the reduced super beam model has 306 DOFs in total, which is much less than the original model. The first six overall natural frequencies and corresponding mode shapes are calculated using the reduced super beam model, and compared with the original model in Table 1. Some minor differences are found in Table 1, and the relative differences in frequencies are less than 3% for almost all modes. Modal assurance criterion (MAC) [9] values of all mode shapes are above 0.88, which indicate the similarity between the reduced and original mode shapes.

Table 1: The frequencies and mode shapes of the cylinder structure using the proposed method

Frequency order	Original model (hz)	Reduced model(hz)	Difference (%)	MAC
1st bending	89.53	84.32	5.82	0.88
1st torsional	130.46	133.72	2.50	0.95
2nd bending	186.59	183.16	1.84	0.90
1st axial	220.86	227.45	2.98	0.91
2nd torsional	261.07	267.59	2.50	0.94
3rd bending	284.84	291.90	2.48	0.89

In model updating, the simulated ‘experimental’ modal data are usually obtained by intentionally introducing damages on some elements, and then the analytical model is updated to identify these damages. In this present paper, the simulated frequencies and mode shapes, which are treated as the ‘experimental’ data, are calculated from the FE model by intentionally reducing the bending rigidity, torsional rigidity and axial rigidity in some parts. The simulated reduction is listed in Table 2 and denoted in figure 3.

Table 2: Assumed rigidity reduction in some parts

	Case1		Case2	
Assumed discrepancy	Part2/Part3 Part6 Part8	(-30%) (-20%) (-30%)	Part1 Part4/Part5 Part9	(-30%) (-20%) (-30%)

The eigensolutions and eigensensitivities of the analytical model are calculated using the proposed reduction method, and match the ‘experimental’ counterparts through an optimization process. The rigidity of all reduced super beam elements is assumed as unknown and chosen as the updating parameter. Accordingly, there are 50 updating parameters in total. The weight coefficients are set to 1.0 for the frequencies and 0.1 for the mode shapes. The Lanczos method is employed to calculate the eigensolutions and the analytical method is used for the eigensensitivities. To calculate the eigensensitivity of the global structure with respect to an updating parameter, the derivative matrices of only one super beam element that contains the parameter is required while those in other substructures are set to zero. The optimization is processed by using Method of Moving Asymptotes (MMA) [12], which stops until the objective change between two successive iterations is less than a specified tolerance 0.05%.

In the case1, the rigidity of randomly selected parts is assumed to be reduced by 30% and 20%, that is, the rigidity of part2, part3 and part8 are reduced by 30% and part6 is reduced by 20%, while the other parts remain unchanged. Then the damaged structure is used to obtain the ‘experimental’ frequencies and mode shapes. The model updating process is conducted to make the analytical model reproduce the ‘experimental’ frequencies and mode shapes. The frequencies and mode shapes before and after the updating are compared in Table 3. It demonstrates that the analytical modal datas closely match the simulated ‘experimental’ counterparts after the updating.

Table 3: The frequencies and mode shapes of the cylinder structure before and after updating (case1)

Frequency order	Experimental frequencies (hz)	Before updating			After updating		
		Analytical frequencies(hz)	Difference (%)	MAC	Analytical frequencies(hz)	Difference (%)	MAC
1st bending	80.82	84.32	4.33	0.85	77.81	3.72	0.89
1st torsional	124.56	133.72	7.35	0.79	119.87	3.76	0.88
2nd bending	167.70	183.16	9.22	0.82	170.88	1.89	0.92
1st axial	211.77	227.45	7.41	0.83	203.39	3.95	0.90
2nd torsional	241.07	267.59	11.01	0.73	233.71	3.05	0.87
3rd bending	264.46	291.90	10.38	0.75	275.46	4.15	0.85

Without losing generality, the rigidity of different parts is assumed to have some known discrepancy as well. In case2, the rigidity of part1, part9 are reduced by 30% and part4, part5 are reduced by 20% (see Table 2). The frequencies and mode shapes before and after the updating are compared in Table 4. In Table 4, the frequencies and mode shapes of the updated model better match the ‘experimental’ counterparts.

It should be noted that using the proposed reduced super beam method, the eigensolutions and eigensensitivities are calculated based on the reduced equation with size of 306×306, rather than on the original global eigenequation with size of 18360×18360. The eigensolutions based on the reduced super beam model takes only 0.25 second, while it takes about 5000 seconds based on the original model. As comparison, the proposed reduced super beam-based model updating method achieves higher efficiency.

Table 4: The frequencies and mode shapes of the cylinder structure before and after updating (case2)

Frequency order	Experimental frequencies (hz)	Before updating			After updating		
		Analytical frequencies(hz)	Difference (%)	MAC	Analytical frequencies(hz)	Difference (%)	MAC
1st bending	82.13	84.32	2.67	0.80	80.07	2.51	0.90
1st torsional	127.01	133.72	5.28	0.78	121.99	3.95	0.89
2nd bending	173.74	183.16	5.42	0.85	176.99	1.87	0.93
1st axial	215.93	227.45	5.34	0.82	207.06	4.11	0.89
2nd torsional	254.11	267.59	5.31	0.83	245.07	3.56	0.91
3rd bending	263.05	291.90	10.97	0.73	274.75	4.45	0.87

7. Conclusions

This paper has proposed a reduced super beam based model updating method for beam-type structure. The eigensolutions and eigensensitivities of the original structure are calculated from a greatly reduced super beam model, and calculation of the eigensensitivities with respect to an updating parameter only requires analysis of the super beam element that contains the parameter. The proposed model updating method is advantageous in improving the computational efficiency. The Application to a typical stiffened cylindrical shell structure demonstrates that the proposed model updating method is efficient to be applied to update large-scale structures with a large number of design parameters.

It is only an introduction of the proposed method in this paper. Although the examples given are simple, which have shown the effectiveness and efficiency. Great efforts will be made to apply the method to more complex practical problems in further research.

8. Acknowledgements

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