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Speed dependent optimisation for variable stiffness vehicle suspension

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Abstract: In this research, an optimization of vehicle suspension performance under different vehicle speeds is studied. Besides finding optimal damping value to achieve a better suspension performance, changing the value of stiffness simultaneously and finding the optimal values in variable stiffness control can achieve the best suspension performance with utilizing the available information of speed. By optimizing the suspension stiffness parameter of quarter-car models subjected to random road excitation with different vehicle speeds, the proposed approach ensures the model to have an optimal operating performance. The optimization method applied in this paper is Genetic Algorithm, which increases the probability of finding the global optimum solution and avoids the convergence to a local minimum. A novel criterion for selecting the optimal suspension parameters is presented in terms of the sprung mass acceleration and the dynamic force degenerated between the wheel and the ground.
Keywords: quarter-car models, genetic algorithm, multi-objective, Magnetorheological damper.

1. Introduction

Suspension is one of the most important units to a vehicle. The parameters of suspension have a great effect to the performance of a car. For conventional passive vehicle suspension with constant parameter value, it is difficult to get good overall performance under different road conditions and speeds. With the development of auto industry and the increase of customer requirement, the research on optimizing vehicle suspension parameters is becoming more and more important. In the suspension parameter optimization progress of traditional vehicle design, the optimization goal, such as riding comfort, suspension deflection and tyre dynamic loading [1], is mostly considered separately.

In recent years, researchers [2, 3] have begun to consider the factors simultaneously, namely, multi-objective optimization. As these criteria mentioned above are conflicting, a suitable multi-objective method with weighting function should be chosen properly. In this case, a multi-objective optimization methodology is applied in this paper, an optimal solution is determined by using the Genetic Algorithm [4]. In order to conduct optimization progress for a vehicle suspension parameter, various suspension models were chosen to simulate, including linear quarter-car model [5], piecewise linear model [2] and sky-hook model [6], etc. In this paper, besides linear quarter-car model, a nonlinear model with Magnetorheological (MR) damper is also chosen to measure. Based on this nonlinear quarter-car model, special attention is paid to investigating the optimal suspension stiffness value under various vehicle horizontal speeds using Genetic Algorithm method.

This paper is organized as follows, the mechanical models employed and the corresponding equations are first presented. Then, the method to create road excitation [7] is introduced. The optimization criteria imposed are outlined and formulated. Next, some typical numerical results are presented. Quarter-car model with MR damper running over roads is also examined. The optimization results are presented under different vehicle speed.

2. Modelling and simulation of suspension under random road profile

2.1. Quarter-Car models

A vehicle suspension is a complex multi-degree freedom vibration system. In order to simplify the model applied, simultaneously, to simulate the running state of a car as realistic as possible, the following assumptions can be applied: (1) The vehicle suspension is a rigid body with a symmetrical structure. (2) The vehicle keeps running at a constant speed. The tyres always keep in contact with road surface. (3) Only vertical vibration should be considered in this case.

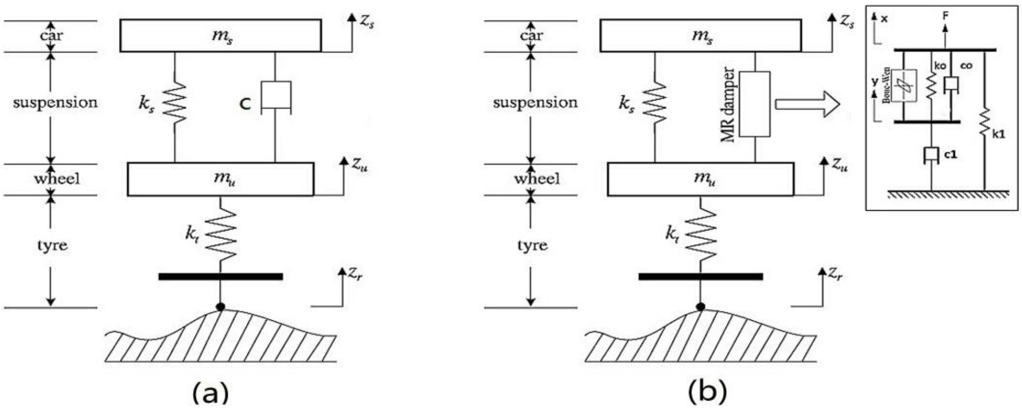

Figure 1: Vehicle model (a) linear model (b) nonlinear model [8]

Based on the assumptions shown above, the vehicle model can be simplified to a two Degree of Freedom (DOF) one. The two degrees of freedom are the sprung mass vertical vibration z_s and the unsprung mass vertical vibration z_u, respectively. The simulation in this article applies two quarter-car models [9], one is a linear model, and the other is a nonlinear model with MR damper. The models are shown in Figure 1. The linear quarter-car dynamic equations can be described by the following differential equations [8]:

$$m_s\ddot{z}_s + c(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = 0 \tag{1}$$

$$m_u\ddot{z}_u + k_t(z_u - z_r) - c(\dot{z}_s - \dot{z}_u) - k_s(z_s - z_u) = 0 \tag{2}$$

In this equation, m_s represents a body vehicle mass (sprung mass), m_u is a wheel vehicle mass (unsprung mass), k_s is the spring stiffness and k_t is the tire stiffness. In addition, z_s is the sprung mass displacement, and z_u represents the unsprung mass displacement. To better predict a MR damper response in model (b), a modified version of the Bouc-Wen model has been proposed by Spencer [10].

2.2. Random road excitation

Road roughness indicates the deflection between road surface and reference plane. Most studies have demonstrated that road roughness is a Gaussian probability distribution with zero mean value. It has smooth traversal characteristic if it is transferred to a stochastic process. The road roughness characteristics can be presented by power spectral density (PSD) function $S_q(\Omega)$. Qualitatively, a larger value of exponent n is defined to describe the roughness at longer wavelengths, while a smaller value at shorter wavelengths. For this reason, a spectra corresponding to the geometrical profile of typical roads can be represented by the following segmented function [6]

$$S_q(\Omega) = \begin{cases} S_q(\Omega_o)(\Omega/\Omega_o)^{-n_1}, & if\ \Omega \leq \Omega_o \\ S_q(\Omega_o)(\Omega/\Omega_o)^{-n_2}, & if\ \Omega \geq \Omega_o \end{cases} \tag{3}$$

Where, Ω is the spatial frequency, $\Omega_o = 1/2\pi$ is a reference spatial frequency, n is the frequency exponent; generally, $n_1 = 2$ and $n_2 = 1.5$ so that the resulting spectrum exhibits a slope discontinuity at $\Omega = \Omega_o$ in a log-log scale. Moreover, the value $S_q(\Omega_o)$ is a power spectral density value under the reference spatial frequencies. For a nonlinear quarter-car model, the road excitation can be generated by the spectral representation method [7, 11], as shown in Eq.(4). Using the harmonic superposition method, the harmonic component under different frequency are added together to generate random road roughness. Supposing that a car is traveling on a given road at a constant speed v, the road irregularities can be simulated by the following formula

$$\mathrm{z}_r(t) = \sum_{i=1}^{n}\left(\sqrt{2S_q(i\Delta\Omega)\Delta\Omega}\right)\sin(i\omega_o t + \varphi_i) \tag{4}$$

In the previous equation, φ_i is the random numbers distributed uniformly among $[0,2\pi]$, where $\Delta\Omega$ is the minimum spatial frequency value we considered, which equals $0.011m^{-1}$. In addition, the value of the fundamental temporal frequency ω_o can be determined by

$$\omega_o = 2\pi\Delta\Omega v \tag{5}$$

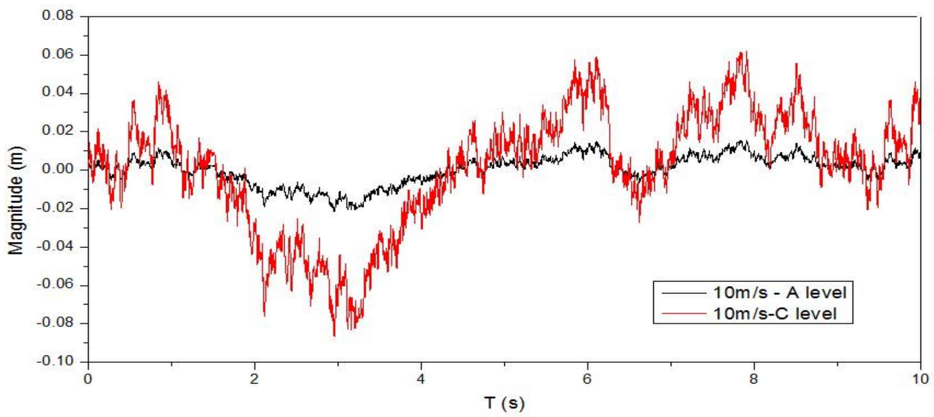

Figure 2: Variation of road roughness of Rank A & C vs time (specific speed:10 m/s)

The random road profile was simulated by Matlab® under different vehicle speeds, between 10 to 40 m/s. The consequence illustrates good-quality road (with $S_q(\Omega_o) = 16 \times 10^{-6}\ m^3$) and bad-quality road (with $S_q(\Omega_o) = 256 \times 10^{-6}\ m^3$) vs. time respectively, shown in Figure 2.

2.3. Suspension performance objective function

To optima the parameter of a vehicle suspension, two factors, riding comfort and tyre deflection, should be considered simultaneous. Particularly, riding comfort, which can be presented by J_1, can be measured by Sprung Mass Acceleration (SMA). Note that in order to normalize the magnitude, SMA should be multiplied by the sprung mass to a force. In addition, another objective function J_2 can be represented by the force developed between the wheel and the ground, namely Tyre Dynamic Load (TDL). Then, the two sub-objective functions are combined into a unified objective function J, as defined by Eq.(6-8). Considering the random characteristic of the road excitation generated in time domain, the following performance index can be determined by expectations

$$\mathrm{J} = w_1 J_1 + w_2 J_2 \tag{6}$$

$$J_1 = E\sqrt{\sum_{i=1}^{N} (m_s \ddot{z}_s)^2 / N} \tag{7}$$

$$J_2 = E\sqrt{\sum_{i=1}^{N} [k_t(z_u - z_r)]^2 / N} \tag{8}$$

where N is the total sampling points. The constants w_1 and w_2 denote the weighting coefficients balancing and adjusting the two performance indices, which include ride comfort and road holding. The proportion is determined by not only the contribution of the individual performance but also the normalized design. Deposing by the aforementioned progress, the weighting coefficients were determined as $w_1 = 0.15$ and $w_2 = 0.85$.

2.4. Simulation progress

In the simulation, the good-quality road profile was chosen as the road excitation. The length of the road is 100m. For the special case of vehicle model with linear properties, the response autospectral density of the linear dynamic system can be obtained easily in frequency domain by applying road profile spectral density under exact vehicle velocity and the stationary vehicle response matrix through the formula [12]

$$S_{XX}(\omega) = H_x(\omega) H_x(-\omega) S_{FF}(\omega) = |H_x(\omega)|^2 S_{FF}(\omega) \tag{9}$$

In the previous equation, $\omega = 2\pi\Delta\Omega v$ is the temporal frequency, $S_{XX}(\omega)$ and $S_{FF}(\omega)$ represent the spectral density of the response and the excitation, respectively. $H_x(\omega)$ is the frequency response functions of the system [13]. Except the linear suspension model presented before, the quarter-car model with MR damper system possesses strong nonlinearities. For this case, Genetic Algorithm (GA), in conjunction with appropriate integration methodologies developed for nonlinear systems [14], is applied to evaluate the suspension response with probabilistic characteristic.

Genetic Algorithms belong to the larger class of evolutionary algorithms (EA), which generate solutions to optimization problems using techniques inspired by natural evolution. This global optimization algorithm has a characteristic that it has less possibility to fall into partial optimal solution in the iterative process. For this reason, it has become a powerful tool to calculate complex optimization issues of a nonlinear system. More detailed discussion and description of Genetic Algorithm are available such as [4]. In the work presented in this article, GADS (Genetic Algorithm and Direct Search Toolbox) in MATLAB® was applied for optimizations. A fixed population size with string length of 50, the generations of 100 are used, crossover fraction is 0.8, and migration fraction is 0.2.

The parameter intending to optimize is the stiffness coefficient of the car suspension. The suspension reference parameter value is taken from the literature [6]. Moreover, the classic stiffness values for the specific method is taken from the literature [14]. The details can be seen in Table 1. More specific, the parameters for the MR damper in the nonlinear model(Figure 1b) were chosen to be $\alpha = 963$ N/cm, $c_o = 53\ N \cdot sec/cm$, $k_o = 14\ N/cm$, $c_1 = 930\ N.sec/cm$, $k_1 = 5.4\ N/cm$, $\gamma = 200\ cm^{-2}$, $\beta = 200\ cm^{-2}$, n = 2, A = 207, and $x_o = 18.9\ cm$.

Table 1: Suspension coefficients for special objective function

Method	$k_s(\mathrm{N}m^{-1})$	$k_t(\mathrm{Ns}m^{-1})$	$\mathrm{c}(\mathrm{Ns}m^{-1})$	m_s(kg)	m_u(kg)
Mixed objective	16000				
SMA	8045	200000	1425	375	60
TD	15836				

3. Numerical results

In terms of a linear model, the suspension response with second moment characteristics in Eq.(4) is readily obtained by the digit sum of the corresponding response of the power spectral density function with specific **ω**. To compare the optimal stiffness values between linear model and nonlinear model, the results are depicted in Figure 3. A major phenomenon in linear model data can be observed that by increasing vehicle's speed, the changing trend of stiffness is getting smaller.

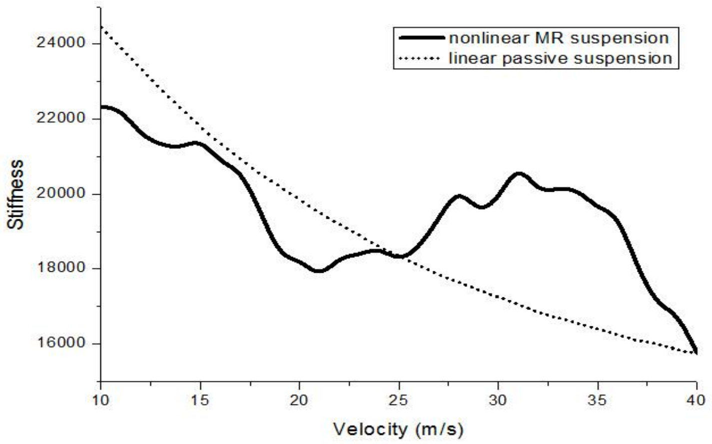

Figure 3: Linear and nonlinear system optimal stiffness values vs. velocity under mixed objective function

In Figure 3, the thick continuous curve illustrates the tendency of nonlinear model optimal parameter, and the thin dash line is obtained from the linear model as a reference. As a consequence, the trend is observed as the dependence of the linear suspension stiffness to the vehicle horizontal velocity, and the value of the nonlinear one demonstrates relative irregular trend within the velocity range considered. The result presented is attributed by the characteristic of MR damper, which has a strong nonlinearity in the relation between the output force and the cylinder's moving velocity.

In order to observe the effect of optimising stiffness value, verification of the result can be done by analysing the performance index of the system using optimal parameters. The normalized performance indexes aforementioned are drawn in Figure 4-6. It can be seen from the figures that, almost at the whole range of vehicle speed considered, the objective function values with optimal stiffness are smaller than the ones with the classic suspension parameter value. More specifically, Figure 4 demonstrates the overall performance of linear suspension.

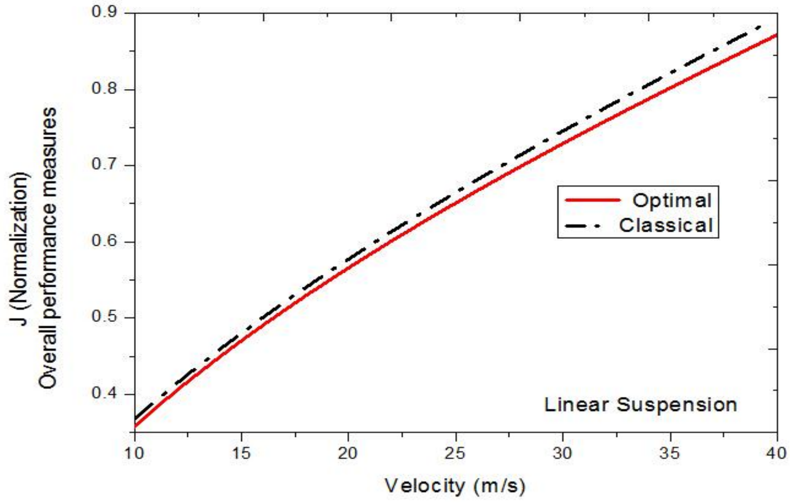

Figure 4: Overall performances for linear model

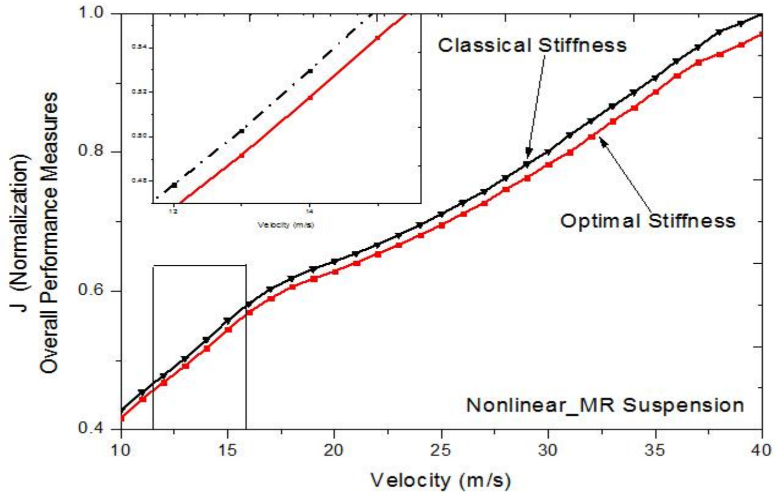

Figure 5: RMS overall performances for nonlinear model

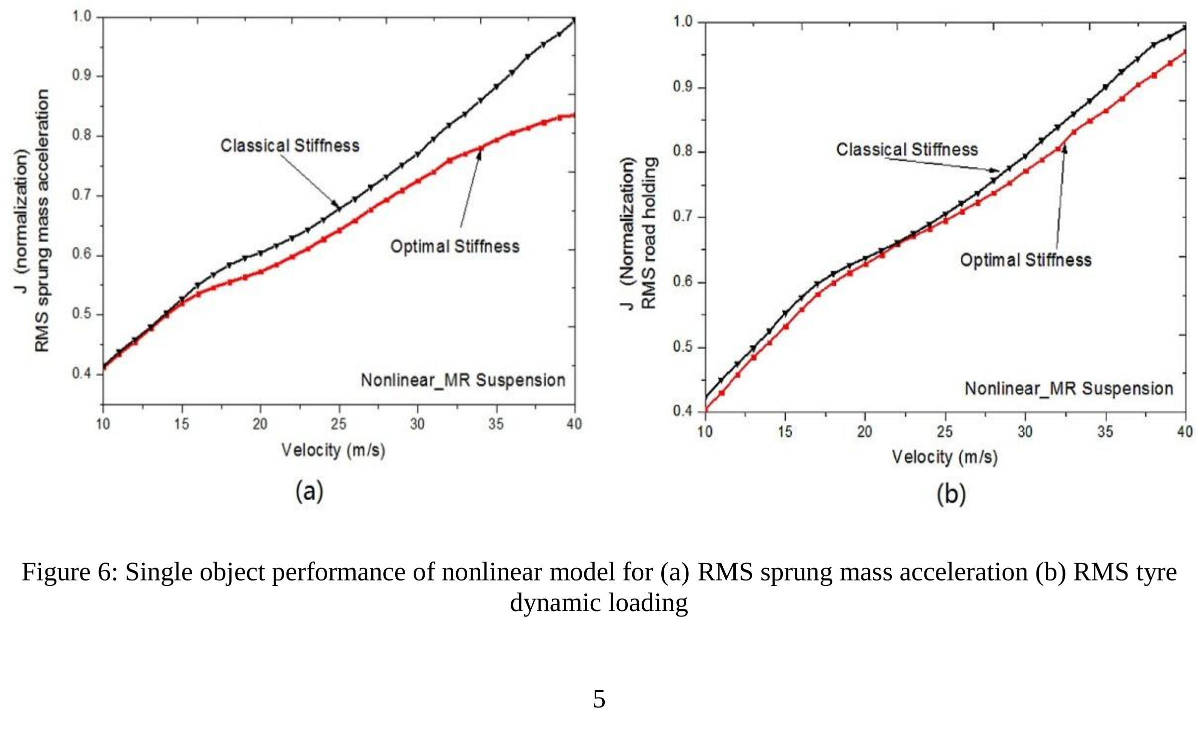

Figure 6: Single object performance of nonlinear model for (a) RMS sprung mass acceleration (b) RMS tyre dynamic loading

As a comparison, Figure 5 represents the nonlinear system overall performance. The associated nonlinear model performance value of sprung mass acceleration and tyre dynamic loading are depicted in Figure 6. For a single objective optimization shown in Figure 6, the overlap of optimal and classic data can be observed under specific vehicle speeds. The overlap effect can be explained after noting that the optimal stiffness value under specific vehicle speeds equals to the classic stiffness value as a reference coincidentally. More specifically, in terms of sprung mass acceleration, overlap is happened in the low vehicle speed region, while the overlap of tyre dynamic loading data can be observed in the medium vehicle speed area. Whether this phenomenon happens or not depends on the reference stiffness value we chosen. In general, after completing vehicle suspension stiffness optimization process, it can be predicted that the suspension performance can be improved in most traveling conditions.

4. Conclusion

Two major factors including improving vehicle comfort and reducing tire dynamic load were considered when optimizing the vehicle suspension. A two degree of freedom quarter-car model was established. Then, the analytical method was applied to solve the linear model problem. Simultaneously, the response of a quarter-car model with MR damper traveling the A level road with constant velocity is considered. The modified Bouc-Wen model is applied to evaluate the hysteretic behaviour of the MR damper. It can be seen from the simulation results that there is an obvious difference between the suspension performance under optimal stiffness value and the one under classic constant stiffness value. Therefore, changing stiffness based on different road profile conditions and the vehicle speeds is meaningful in practical application. Further study on real-time control of stiffness and damping based on the developed semi-active suspension will be considered and tested in the next step.

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