# Weight minimization of trusses with natural frequency constraints

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**1. Abstract**

The article presents the optimization of trusses subject to natural frequency constraints. Three planar trusses (10-bar, 37-bar and 200-bar structures) and three spatial trusses (52-bar, 72-bar and 120-bar structures) are surveyed. Differential Evolution method combined with finite element code is employed to find the optimal cross-section sizes and node coordinates. Constraint-handling technique is only based on a clear distinction between feasible solutions and infeasible solutions. The obtained results are equivalent to or better than solutions in literature.
**2. Keywords:** Truss optimization, natural frequency, differential evolution

## 3. Introduction

Truss optimization is one of the oldest developments of structural optimization. There were many publications on this field. However, except for simple problems that have been solved analytically, the search space and constraints are quite complex in most of actual structures. Consequently, there might not have explicit functions of constraints and closed-form solutions of stress, strain or natural frequency for these structures. In addition, due to the efficiency and capability of optimization methods, the so-called "optimum result" is often revised and improved with various approaches and algorithms. Structural optimization can be classified into three categories: cross-section size, geometry and topology. Depending on requirements, the study can include from one to all of the above types. This article only considers two types of optimization: the cross-section size and the geometry of trusses with natural frequency constraints. Six planar and spatial trusses are surveyed in this work. These structures are very common in literature and there are similar studies in [1-6]. However, the differences in this article are the quality of results and new groups of elements in 200-bar and 52-bar structures.

Although Differential Evolution (DE) has been proposed by Storn and Price [7] since 1997 and attracted many studies, there is not much application of DE to truss optimization with natural frequency constraints. Kaveh and Zolghadr [5] have compared nine algorithms but without DE. Recently, Pholdee and Bureerat [6] have considered DE in their comparison of various meta-heuristic algorithms for these problems.

## 4. Differential Evolution

Differential Evolution is a non-gradient optimization method which utilizes information within the vector population to adjust the search direction. Consider D-dimensional vectors $x_{i,G}$, $i = 1, 2,..., N$ as a population for each generation G, $x_{i,G+1}$ as a mutant vector in generation $(G + 1)$ and $u_{i,G+1}$ as a trial vector in generation $(G + 1)$. There are three operators in DE as follows.

- Mutation:

$$v_{i,G+1} = x_{r1,G} + FM(x_{r2,G} - x_{r3,G}) \quad i = 1, 2,\dots, N \tag{1a}$$

Another variant uses the best vector and two difference vectors for mutation. In this variant, Eq. (1a) becomes

$$v_{i,G+1} = x_{best,G} + FM(x_{r1,G} + x_{r2,G} - x_{r3,G} - x_{r4,G}) \tag{1b}$$

where $r_1$, $r_2$, $r_3$, $r_4$ are random numbers in [1, $N$] and integer, which are mutually different and different from the running index $i$; and $FM$ is mutation constant in [0, 2].

In this research, the variant with Eq. (1b) will be adopted.

- Crossover:

$$u_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \dots, u_{Di,G+1}) \tag{2}$$

where $u_{ji,G+1} = \begin{cases} v_{ji,G+1} & if\,(r \le CR)\; or\, j = k \\ x_{ji,G} & if\,(r > CR)\; and\, j \ne k \end{cases}, \quad j = 1, 2,...D$

where $CR$ is the crossover constant in [0, 1]; $r$ is random number in (0, 1); and $k$ is random integer number in [1, D], which ensures that $u_{i,G+1}$ gets at least one component from $v_{i,G+1}$.

- Selection:

$$x_{i,G+1} = \begin{cases} u_{i,G+1} & if\, f(u_{i,G+1}) < f(x_{i,G}) \\ x_{i,G} & otherwise \end{cases} \quad (3)$$

where $f$ is the objective function.

The DE was originally proposed for unconstrained optimization problems. For constrained optimization problems, a constraint-handling technique suggested by Jimenez et al [8] is supplemented. In this technique, the comparison of two solutions complies with the rule: *(i)* for two feasible solutions, the one with a better objective function value is chosen; *(ii)* for one feasible solution and one infeasible solution, the feasible solution is chosen; and *(iii)* for two infeasible solutions, the one with a smaller constraint violation is chosen. This technique requires no additional coefficient and gives a natural approach to the feasible zone from trials in the infeasible zone.

**5. Problem Formulation and Results**

Generally, the optimization problem can be defined as follows:

$$\text{Minimizing} \quad W = \sum_{i=1}^{N} \rho_i L_i A_i \quad (4)$$

$$\text{Subject to:} \quad \omega_i \geq \omega_{i,min} \quad i = 1..K$$
$$A_{i,min} \leq A_i \leq A_{i,max} \quad i = 1..N$$
$$x_{i,min} \leq x_i \leq x_{i,max} \quad i = 1..M$$

where, $W$ is the truss weight; $\rho_i$, $L_i$ and $A_i$ are the density, length and cross-section area of the $i^{th}$ element, respectively; $\omega_i$ and $\omega_{i,min}$ are the $i^{th}$ natural frequency and corresponding frequency limit; $A_{i,min}$ and $A_{i,max}$ are the lower bound and upper bound of the $i^{th}$ cross-section area; $x_{i,min}$ and $x_{i,max}$ are the lower bound and upper bound of the $i^{th}$ coordinate; $N$, $K$ and $M$ are the number of elements in the truss, number of frequencies subject to limits and number of nodes subject to coordinate constraints, respectively.

This study surveys three planar trusses (10-bar, 37-bar and 200-bar structures) and three spatial trusses (52-bar, 72-bar and 120-bar structures). For brevity, the basic parameters of problems and the optimal results compared with those in the literature are listed in table 1. Parameters of DE are as follows: population N = 50 (except N = 150 for 200-bar truss), crossover constant CR = 0.9, mutation constant FM = 0.5, number of iterations I = 150. Each problem is performed in 50 independent runs (except 100 runs for 200-bar truss). Other details are described in each problem.

Table 1: Parameters and the minimal weight of trusses

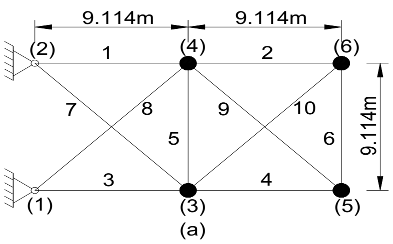
| Parameters | Unit | Data of problems | | | | | |
|---|---|---|---|---|---|---|---|
| | | 10-bar | 37-bar | 200-bar | 52-bar | 72-bar | 120-bar |
| Modulus of elasticity, E | $N/m^2$ | $6.98\times10^{10}$ | $2.1\times10^{11}$ | $2.1\times10^{11}$ | $2.1\times10^{11}$ | $6.98\times10^{10}$ | $2.1\times10^{11}$ |
| Material density , ρ | $kg/m^3$ | 2770 | 7800 | 7860 | 7800 | 2770 | 7971.81 |
| Frequencies constraints | Hz | $\omega_1 \geq 7$<br>$\omega_2 \geq 15$<br>$\omega_3 \geq 20$ | $\omega_1 \geq 20$<br>$\omega_2 \geq 40$<br>$\omega_3 \geq 60$ | $\omega_1 \geq 5$<br>$\omega_2 \geq 10$<br>$\omega_3 \geq 15$ | $\omega_1 \leq 15.916$<br>$\omega_2 \geq 28.648$ | $\omega_1 = 4$<br>$\omega_3 \geq 6$ | $\omega_1 \geq 9$<br>$\omega_2 \geq 11$ |
| Cross-section bounds | $m^2$ | $[0.645\times10^{-4}, 40\times10^{-4}]$ | $[10^{-4},10^{-3}]$ | $[0.1\times10^{-4}, 30\times10^{-4}]$ | $[10^{-4}, 10^{-3}]$ | $[0.645\times10^{-4}, 30\times10^{-4}]$ | $[10^{-4}, 129.3\times10^{-4}]$ |
| Nodes coordinate bounds | m | n/a | See details in 5.2 | n/a | See details in 5.4 | n/a | n/a |
| Minimal weight $W_{min}$ | kg | **524.56** | **359.45** | **2296.38** | **191.28** | **324.36** | **8710.90** |
| $W_{min}$ [1] | kg | 529.09 | n/a | 2298.61 | 197.31 | 327.51 | 9046.34 |
| $W_{min}$ [2] | kg | 535.61 | n/a | n/a | n/a | 326.67 | n/a |
| $W_{min}$ [3] | kg | 524.88 | 364.72 | n/a | 193.36 | 324.50 | n/a |
| $W_{min}$ [4] | kg | 535.14 | 363.03 | n/a | 207.27 | n/a | n/a |
| $W_{min}$ [5] | kg | 532.34 | 360.56 | n/a | 195.62 | 334.66 | 8886.92 |
| $W_{min}$ [6] | kg | 524.49 | 359.25 | n/a | 195.19 | 324.32 | n/a |

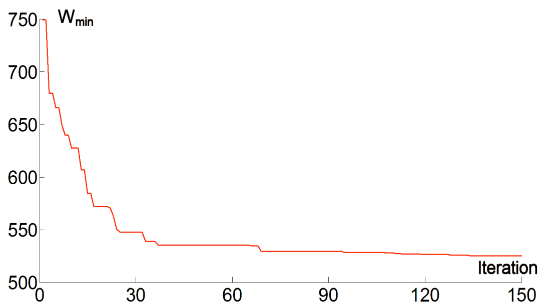
Table 2: Optimal results of planar trusses

| 10-bar truss | | 37-bar truss | | 200-bar truss | |
|---|---|---|---|---|---|
| **Parameters** | **Results** | **Parameters** | **Results** | **Parameters** | **Results** |
| $A_1$ ($cm^2$) | 35.0930 | $Y_3$,$Y_{19}$ (m) | 0.9736 | $A_1$ ($cm^2$) | 0.2789 |
| $A_2$ ($cm^2$) | 14.8802 | $Y_5$,$Y_{17}$ (m) | 1.3372 | $A_2$ ($cm^2$) | 0.2725 |
| $A_3$ ($cm^2$) | 35.3906 | $Y_7$,$Y_{15}$ (m) | 1.4965 | $A_3$ ($cm^2$) | 5.7222 |
| $A_4$ ($cm^2$) | 14.7917 | $Y_9$,$Y_{13}$ (m) | 1.6024 | $A_4$ ($cm^2$) | 0.5423 |
| $A_5$ ($cm^2$) | 0.6450 | $Y_{11}$ (m) | 1.6834 | $A_5$ ($cm^2$) | 1.4830 |
| $A_6$ ($cm^2$) | 4.5550 | $A_1$, $A_{27}$ ($cm^2$) | 2.9844 | $A_6$ ($cm^2$) | 3.1676 |
| $A_7$ ($cm^2$) | 23.6841 | $A_2$, $A_{26}$ ($cm^2$) | 1.0570 | $A_7$ ($cm^2$) | 4.8800 |
| $A_8$ ($cm^2$) | 23.5333 | $A_3$, $A_{24}$ ($cm^2$) | 1.0806 | $A_8$ ($cm^2$) | 7.9840 |
| $A_9$ ($cm^2$) | 12.5161 | $A_4$, $A_{25}$ ($cm^2$) | 2.6508 | $A_9$ ($cm^2$) | 18.7813 |
| $A_{10}$ ($cm^2$) | 12.2117 | $A_5$, $A_{23}$ ($cm^2$) | 1.3097 | $A_{10}$ ($cm^2$) | 0.1000 |
| $\omega_1$ (Hz) | 7.0002 | $A_6$, $A_{21}$ ($cm^2$) | 1.0469 | $A_{11}$ ($cm^2$) | 0.1000 |
| $\omega_2$ (Hz) | 16.2058 | $A_7$, $A_{22}$ ($cm^2$) | 2.6815 | $A_{12}$ ($cm^2$) | 0.1000 |
| $\omega_3$ (Hz) | 20.0003 | $A_8$, $A_{20}$ ($cm^2$) | 1.3132 | $A_{13}$ ($cm^2$) | 0.1000 |
| $W_{min}$ (kg) | 524.56 | $A_9$, $A_{18}$ ($cm^2$) | 1.4278 | $A_{14}$ ($cm^2$) | 0.2326 |
| | | $A_{10}$, $A_{19}$ ($cm^2$) | 2.6826 | $A_{15}$ ($cm^2$) | 0.8481 |
| | | $A_{11}$, $A_{17}$ ($cm^2$) | 1.1798 | $A_{16}$ ($cm^2$) | 1.2137 |
| | | $A_{12}$, $A_{15}$ ($cm^2$) | 1.2232 | $A_{17}$ ($cm^2$) | 1.6352 |
| | | $A_{13}$, $A_{16}$ ($cm^2$) | 2.3931 | $A_{18}$ ($cm^2$) | 2.0476 |
| | | $A_{14}$ ($cm^2$) | 1.0000 | $A_{19}$ ($cm^2$) | 4.3721 |
| | | $\omega_1$ (Hz) | 20.0565 | $\omega_1$ (Hz) | 5.0004 |
| | | $\omega_2$ (Hz) | 40.0129 | $\omega_2$ (Hz) | 12.3446 |
| | | $\omega_3$ (Hz) | 60.0983 | $\omega_3$ (Hz) | 15.0018 |
| | | $W_{min}$ (kg) | 359.45 | $W_{min}$ (kg) | 2296.38 |

**5.1 Planar 10-bar truss**

The truss is shown in figure 1 where the added mass of 454 kg is attached at nodes 3– 6. Only cross-section sizes are optimized in this problem. The optimal weight is $W_{min}$ = 524.56 kg and converges after 150 iterations. Sizes of optimal cross-sections and natural frequencies are presented in table 2.




(b)

Figure 1: (a) Model of 10-bar truss
(b) Optimal cross-section sizes

**5.2 Planar 37-bar truss**

The truss is shown in figure 2 where the added mass of 10 kg is attached at lower nodes. The lower bars have a fixed cross-section area of 0. 004$m^2$. For this problem, both cross-section sizes and y-coordinate of upper nodes are optimized. Upper nodes can move symmetrically in vertical direction. The optimal weight is $W_{min}$ = 359.45 kg and converges after 150 iterations. Sizes of optimal cross-sections and coordinates of nodes are listed in table 2.

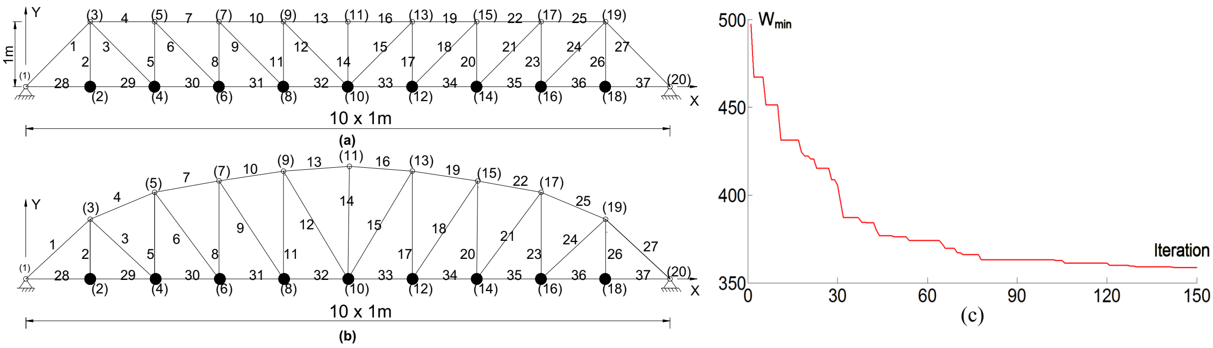
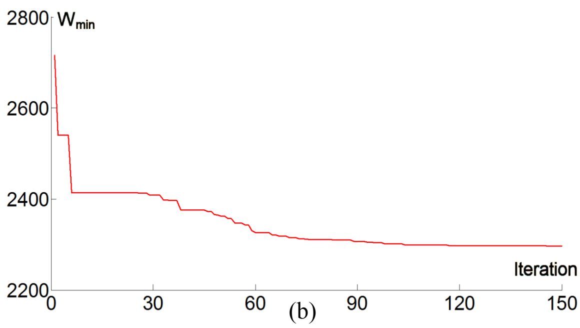



Figure 2: (a) Model of initial 37-bar truss, (b) Optimal shape,
(c) Convergence of the optimal result

### 5.3 Planar 200-bar truss

The truss is shown in figure 3 where the added mass of 100 kg is attached at nodes 1–5. Only cross-section sizes are optimized. In comparison with the others, this problem has the high static indeterminacy, large number of element and wide range of cross-section size. This makes the optimization more difficult. Our approach is to use 19 groups of elements instead of 29 groups in [1]. Thus, the search space decreases one third.

For this structure, it notes generally that the lower storey, the bigger cross-section size. For example, the cross-section sizes gradually increase in groups of elements (1, 2, 3), (4, 5, 6, 7, 8, 9), (10, 11, 12, 13, 14), (15, 16, 17, 18, 19). In the article, we use this characteristic for the initialization of random values in DE algorithm. The optimal weight is $W_{min}$ = 2296.38 kg. Sizes of optimal cross-sections and natural frequencies are presented in table 2.



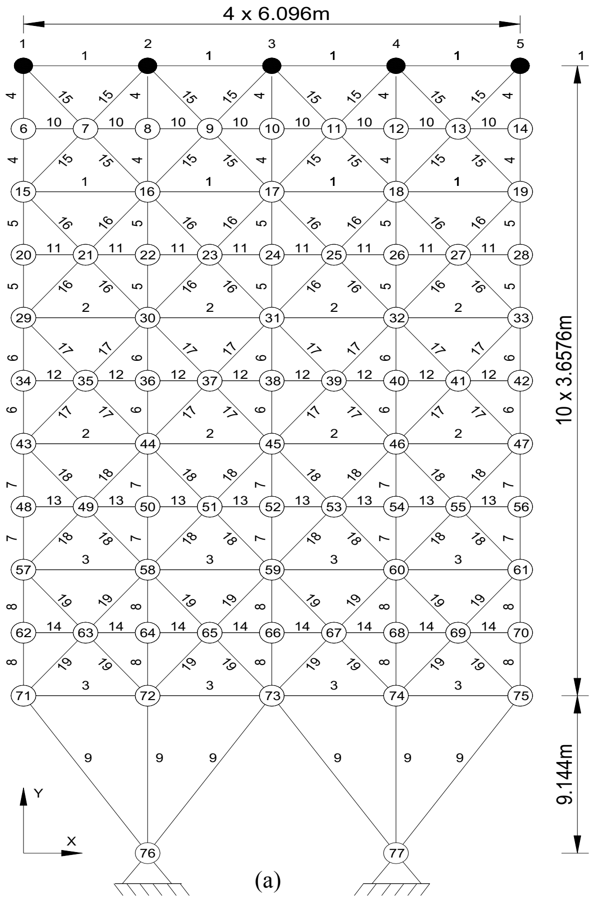




Figure 3: (a) Model of 200-bar truss
(b) Convergence of the optimal result

### 5.4 Spatial 52-bar truss

The truss is shown in figure 4 where the added mass of 50kg is attached at free nodes 1–13. Both cross-section sizes and coordinate of nodes are optimized in this problem. Free nodes can move in range of ±2m from their initial positions such that the symmetry is reserved. It notes that elements are divided to 9 groups which differs from 8 groups in [1]. The optimal weight is $W_{min}$ = 191.28 kg and converges after 150 iterations. Sizes of optimal cross-sections, coordinates of nodes and natural frequencies are listed in table 3.

### 5.5 Spatial 72-bar truss

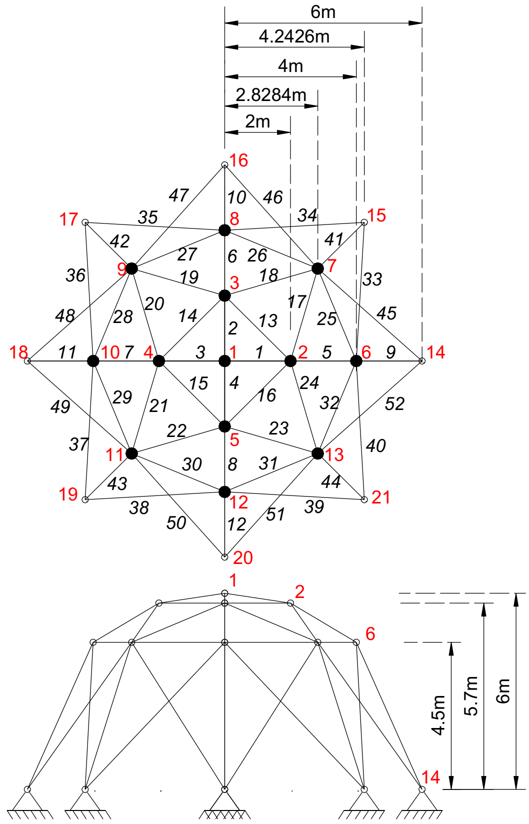
The truss is shown in figure 5 where the added mass of 2270 kg is attached at nodes 17–20. Only cross-section sizes are optimized in this problem. Elements are divided to 16 groups, as seen in table 3. The optimal weight is $W_{min}$ = 324.36 kg and converges after 150 iterations. Sizes of optimal cross-sections and natural frequencies are listed in table 3.

### 5.6 Spatial 120-bar truss

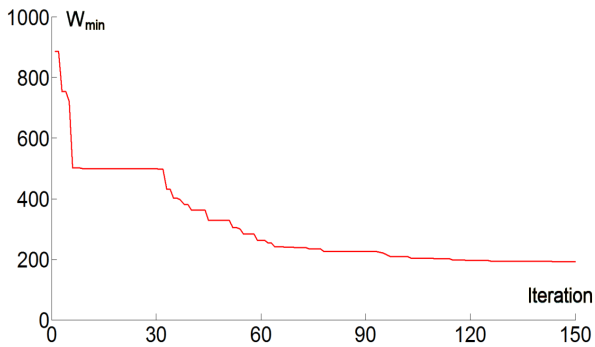
The truss is shown in figure 6 where the added masses are attached at nodes as follows: 3000 kg at node 1, 500 kg at nodes 2– 3, and 100 kg at nodes 14–37. Elements are divided to 7 groups, as seen in table 3. The optimal weight is $W_{min}$ = 8710.90 kg and converges after 150 iterations. Sizes of optimal cross-sections and natural frequencies are listed in table 3.

## 6. Conclusion

For six considered problems, Differential Evolution gives better or equivalent results compared with other methods in the literature. The convergence is rapid in round first 50 iterations. Especially, the different number of element groups in 52-bar truss and 200-bar truss also makes the minimum weights improved.



(a)

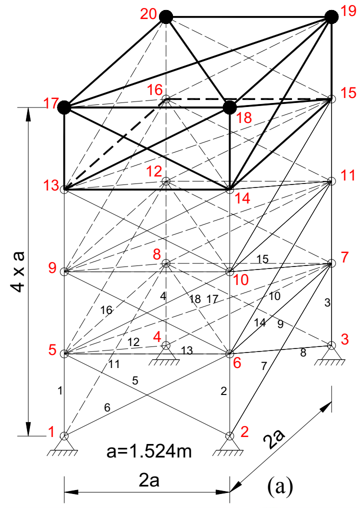


(b)

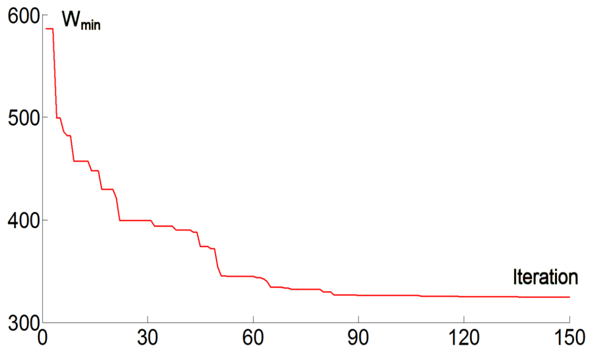
Figure 4: (a) Model of initial 52-bar truss
(b) Convergence of the optimal result

Table 3: Optimal results of spatial trusses

| 52-bar truss | | 72-bar truss | | 120-bar truss | |
|---|---|---|---|---|---|
| Parameters | Results | Parameters | Results | Parameters | Results |
| $A_1 – A_4$ (cm$^2$) | 1.0000 | $A_1 – A_4$ (cm$^2$) | 16.9972 | $A_1 – A_{12}$ (cm$^2$) | 19.4686 |
| $A_5 – A_8$ (cm$^2$) | 1.5406 | $A_5 – A_{12}$ (cm$^2$) | 7.9173 | $A_{13} – A_{24}$ (cm$^2$) | 40.6526 |
| $A_9 – A_{12}$ (cm$^2$) | 1.0889 | $A_{13} – A_{16}$ (cm$^2$) | 0.6450 | $A_{25} – A_{36}$ (cm$^2$) | 10.7573 |
| $A_{13} – A_{16}$ (cm$^2$) | 1.2652 | $A_{17} – A_{18}$ (cm$^2$) | 0.6503 | $A_{37} – A_{60}$ (cm$^2$) | 21.0849 |
| $A_{17} – A_{24}$ (cm$^2$) | 1.0339 | $A_{19} – A_{22}$ (cm$^2$) | 12.5286 | $A_{61} – A_{84}$ (cm$^2$) | 9.7139 |
| $A_{25} – A_{32}$ (cm$^2$) | 1.2461 | $A_{23} – A_{30}$ (cm$^2$) | 8.0249 | $A_{85} – A_{96}$ (cm$^2$) | 11.6176 |
| $A_{33} – A_{40}$ (cm$^2$) | 1.3722 | $A_{31} – A_{34}$ (cm$^2$) | 0.6454 | $A_{97} – A_{120}$ (cm$^2$) | 14.8947 |
| $A_{41} – A_{44}$ (cm$^2$) | 1.0000 | $A_{35} – A_{36}$ (cm$^2$) | 0.6450 | $\omega_1$ (Hz) | 9.0013 |
| $A_{45} – A_{52}$ (cm$^2$) | 1.6035 | $A_{37} – A_{40}$ (cm$^2$) | 7.9920 | $\omega_2$ (Hz) | 11.0001 |
| $Z_1$ (m) | 6.0710 | $A_{41} – A_{48}$ (cm$^2$) | 8.0029 | $W_{min}$ (kg) | 8710.90 |
| $X_2$ (m) | 2.6047 | $A_{49} – A_{52}$ (cm$^2$) | 0.6450 | | |
| $Z_2$ (m) | 3.7267 | $A_{53} – A_{54}$ (cm$^2$) | 0.6479 | | |
| $X_6$ (m) | 4.1584 | $A_{55} – A_{58}$ (cm$^2$) | 3.6988 | | |
| $Z_6$ (m) | 2.5000 | $A_{59} – A_{66}$ (cm$^2$) | 7.8182 | | |
| $\omega_1$ (Hz) | 15.2070 | $A_{67} – A_{70}$ (cm$^2$) | 0.6450 | | |
| $\omega_2$ (Hz) | 28.6487 | $A_{71} – A_{72}$ (cm$^2$) | 0.6450 | | |
| $W_{min}$ (kg) | 191.28 | $\omega_1$ (Hz) | 4.0002 | | |
| | | $\omega_3$ (Hz) | 6.0008 | | |
| | | $W_{min}$ (kg) | 324.36 | | |

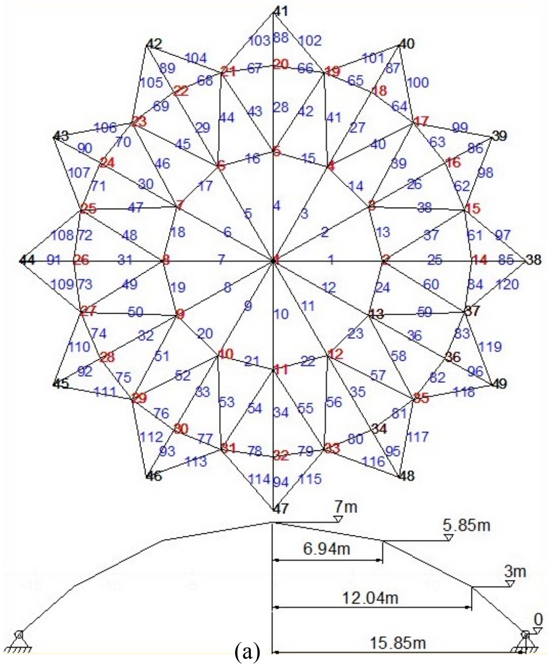


(a)

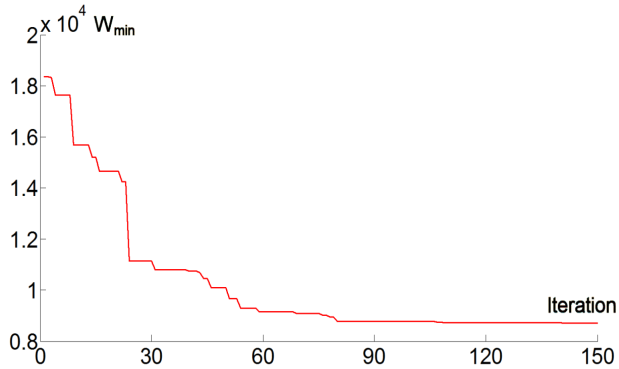


(b)

Figure 5: (a) Model of 72-bar truss
(b) Convergence of the optimal result



(a)



(b)

Figure 6: (a) Model of 120-bar truss,
(b) Convergence of the optimal result

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