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A Sensitivity-based Coordination Method for Optimization of Product Families

Jun Zou[1], Wei-Xing Yao[2], Jun-Feng Zheng[3]

[1] Nanjing University of Aeronautics and Astronautics, Nanjing, China, sc.zoujun@gmail.com
[2] Nanjing University of Aeronautics and Astronautics, Nanjing, China, wxyao@nuaa.edu.cn
[3] Nanjing University of Aeronautics and Astronautics, Nanjing, China, zjfbest@nuaa.edu.cn

1. Abstract
This article discusses the bi-level decomposition approach for the optimization of product families with predefined platforms, and the challenge lies in providing an optimal compromise between the competing needs of all family members. To improve the efficiency of the optimization process, a new sensitivity-based coordination method (SCM) is proposed. The key idea in this approach is that the system coordinator not only provides consistent shared variables, but also makes tradeoff between all the products by using of sensitivity information. The coordinated shared variables are determined by minimizing performance deviation with respect to the optimal solution of subproblems and constraints violation. Each subproblem owns a significant degree of independence and can be solved in a simultaneous way. The numerical performance of the proposed method is investigated, and the results suggest that the new approach is robust and leads to a substantial reduction in computational effort compared with analytical target cascading (ATC) method. Then the proposed methodology is successfully applied to the structural optimization problem of wing structures for an unmanned aircraft family, which is more complicated and related to practical implementation issues.
2. Keywords: product family optimization; bi-level decomposition; sensitivity; system coordination; structural optimization

3. Introduction
Due to the highly competitive global marketplace, the contradiction between product variety and development and production costs is prominent. Under this backdrop, the product family design has received considerable attention from both industry [1] and academia [2, 3] in recent years.
In general terms, a product family is a group of related products that share a collection of the common elements, which called product platform, to satisfy a variety of market niches [4]. The core technology of product family design is product platform. Meyer and Lehnerd [5] defined the product platform as the set of parts, interfaces, and manufacturing processes that are shared among a set of products and allow the development of derivative products with cost and time savings. Jiao *et al.* [6] and Simpson *et al.* [7, 8] summarized the relevant terminology and applications in detail, respectively.
The challenge when optimizing a family of products with predefined platforms lies in providing an optimal compromise between the competing needs of all family members [9]. The product family problem can be solved by one-stage or multi-stage approach. When a product family problem is relatively small, the one-stage approach is preferred, as it can yield the best overall performance of the product family because the optimization is not partitioned into two or more stages [10]. The dimensionality of one-stage optimization problems is, however, considerably higher than in multi-stage approaches and often leads to computational difficulties. While for the decomposition-based multi-stage approach, in which the family design problem is partitioned into smaller subproblems, the complexity increases only linearly with the number of individual products and is more suitable for design of large and complex product families [11].
This article discusses the multi-stage approach for the optimization of product families with predefined platforms. Fellini *et al.* [12] pointed out that developing a rigorous global coordination strategy is necessary to increase the efficiency and robustness for product family design. Simpson *et al.* [13] adopted a two-stage approach that the product platform is designed during the first stage of the optimization, followed by instantiation of the individual products during the second stage. Allison *et al.* [14] decomposed the family design problem by individual variant, and the shared variables were coordinated at system level. Then he applied two decomposition-based method, collaborative optimization (CO) and ATC, to the aircraft family problem.
Some scholars have proposed some new methods to improve the computation efficiency of product family optimization. Kokkolaras *et al.* [15] presented an extended ATC methodology for product family problem. The ATC formulation for a single product was extended to a family of products to accommodate the presence of a shared product platform and local design targets. Roth and Kroo [16] presented a distributed optimization based on CO and ATC, named Enhanced Collaborative Optimization (ECO). The system level optimum is simply the average of the responses returned from the subspaces, and subspace’s constraint set includes local constraints

and models of constraints from other subspaces. Öman and Nilsson [17] presented a multi-stage method named Critical Constraint Method (CCM) for structural optimization of product family problem, in which the problem reduction is performed by only considering the constraints that are critical in the optimal solution.
To improve the efficiency of the system coordinator and reduce the number of iterations needed for convergence, in this article a new SCM method is proposed. The key idea in this approach is that the system coordinator not only provides consistent shared variables, but also makes tradeoff between all individual products by using of sensitivity information to evaluate the performance deviation and constraint violation resulting from sharing. The new algorithm is described and tested by a numerical test case and a family of aircraft wing structures design example in this paper.

4. Problem formulation

The family design problem with predefined platforms is formulated as:

$$\begin{aligned} &\text{find} \quad (\boldsymbol{x}_s, \boldsymbol{x}_{li}) \\ &\min \quad F(f_1(\boldsymbol{x}_s, \boldsymbol{x}_{l1}), \cdots, f_n(\boldsymbol{x}_s, \boldsymbol{x}_{ln})) \\ &\text{s.t.} \quad \boldsymbol{g}_i(\boldsymbol{x}_s, \boldsymbol{x}_{li}) \le \boldsymbol{0} \quad (i = 1, 2, \cdots, n) \end{aligned} \tag{1}$$

where $\boldsymbol{x}_s$ is the vector of shared variables, which are the design variables of platform and shared between all the family members. $\boldsymbol{x}_{li}$ is the vector of individual local variables for *i*th product. $f_i(\boldsymbol{x}_s, \boldsymbol{x}_{li})$ and $\boldsymbol{g}_i(\boldsymbol{x})$ is the design objective and set of constraints for *i*th product respectively. *F* is the design objective of the whole product family, which is the function of $f_i(\boldsymbol{x}_s, \boldsymbol{x}_{li})$. The product family includes *n* product variants.

5. Sensitivity-based coordination method

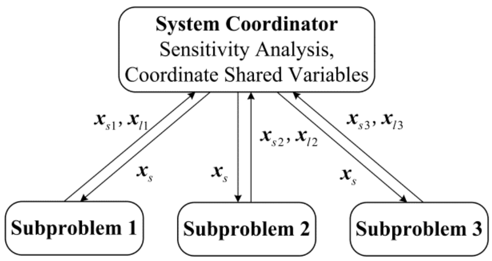


Figure 1: Schematic illustration of SCM

The basic idea of the new method is to provide an efficient system coordinator, so that the optimization process can converge with less number of iterations. This is done by decomposing the family problem into several subproblems and one system coordinator. Each subproblem is responsible for specifying the variables for one family member, and the task of the system level is to coordinate the different design of shared variables obtained from subproblems based on sensitivity information. The proposed SCM to solve the product family optimization problem defined in Eq. (1) is illustrated in Figure 1 and is here described in detail below.

5.1. System coordinator

The objective of system level is to ensure that all subsystems use the same values of shared variables $\boldsymbol{x}_s$. As each subsystem has very limited knowledge of the status and preferences of the other subsystems, a system coordinator with global sense will greatly improve the optimization efficiency. In general, sharing may cause deviations from the individually optimized design of subsystems, and inapposite consistency coordination may lead to infeasible design or large performance loss for the product variants [18]. The optimal value of coordinated shared variables $\boldsymbol{x}_s$ is determined by minimizing the performance deviation with respect to the optimal solution of subproblems while remaining in feasible space.
Formally, the system level is defined as:

$$\begin{aligned} &\text{find} \quad \boldsymbol{x}_s \\ &\min \quad \sum_{i=1}^{n} \left| F^{i,*} - F^{i,o} \right| + \sum_{i=1}^{n} \sum_{g_{ij} \in \boldsymbol{g}_i} \max(g_{ij}^*, 0) \end{aligned} \tag{2}$$

where $F^{i,o}$ corresponds to the value of *F* obtained from the solution of *i*th subproblem, as described in next part, while $F^{i,*}$ and g_{ij}^* $(g_{ij} \in \boldsymbol{g}_i)$ is the value of *F* and constraint evaluated at a consistent shared variables $\boldsymbol{x}_s$.
It can be seen that the system level is an unconstrained minimization problem. A first order Taylor series approximation is introduced to evaluate the value of performance deviation and constraints. That is:

$$F^{i,*} \approx (\nabla F^{si,o})^T(\boldsymbol{x}_s - \boldsymbol{x}_{si}^o) + F^{i,o}(\boldsymbol{x}_{si}^o, \boldsymbol{x}_{li}^o)$$
$$g_{ij}^* \approx (\nabla g_{ij}^{si,o})^T(\boldsymbol{x}_s - \boldsymbol{x}_{is}^o) + g_{ij}^o(\boldsymbol{x}_{si}^o, \boldsymbol{x}_{li}^o) \tag{3}$$

where $\boldsymbol{x}_s$ is the vector of shared variables after coordination, and $(\boldsymbol{x}_{si}^o, \boldsymbol{x}_{li}^o)$ is the optimal solution of *i*th subproblem. $\nabla F^{si,o}$ and $\nabla g_{ij}^{si,o}$ is the gradient of *F* and g_{ij} evaluated at the optimal design point of subproblem *i*. It easily to find out that the values of shared variables $\boldsymbol{x}_s$ attempt to be closer to the solution of the subproblem that objective *F* and constraints are more sensitive to the change of shared variables.

5.2. Subproblem optimization

Each subspace optimization problem is responsible for the design of one individual product variant, which includes both shared variables $\boldsymbol{x}_{si}$ and individual variables $\boldsymbol{x}_{li}$. The formulation of subsystem problem, as illustrated for the *i*th subproblem, is shown as follows:

$$\begin{aligned} &\text{find} \quad (\boldsymbol{x}_{si}, \boldsymbol{x}_{li}) \\ &\min \quad F(f_1, \cdots, f_i(\boldsymbol{x}_{si}, \boldsymbol{x}_{li}), \cdots, f_n) + \left\| \boldsymbol{w}_i \circ (\boldsymbol{x}_s - \boldsymbol{x}_{si}) \right\|_2^2 \\ &\text{s.t.} \quad \boldsymbol{g}_i(\boldsymbol{x}_{si}, \boldsymbol{x}_{li}) \le \boldsymbol{0} \end{aligned} \tag{4}$$

where $\boldsymbol{w}_i$ is the vector of penalty weights. The symbol $\circ$ is used to indicate term-by-term multiplication of vectors. The sequence of $\boldsymbol{w}$ is nondecreasing to guide the optimization process to convergence.

The system coordinator provides targets for shared variables $\boldsymbol{x}_s$ and the objective value f_j $(j \neq i)$ at the optimal solution of the other subproblems, which are treated as parameters. The objective of subproblem is a combination of the product family objective and a compatibility term.

5.3. Stopping criteria

The optimization is stopped based on two conditions: The change of the objective value and inconsistency of shared variables have to be smaller than defined corresponding critical values.

$$\left| \frac{F^{(\mathrm{k})} - F^{(\mathrm{k}-1)}}{F^{(\mathrm{k}-1)}} \right| \le \varepsilon_F \quad \text{AND} \quad \max(\left\| \boldsymbol{x}_s - \boldsymbol{x}_{is} \right\|_2, i = 1, \cdots, n) \le \varepsilon_x \tag{5}$$

where *F* is the value of objective function at the optimal point of subproblem optimizations, and ε_F is the corresponding stop criterion. $\boldsymbol{x}_{is}$ refers to the optimal solution of shared variables at subproblem *i*, and $\boldsymbol{x}_s$ refers to the vector of shared variables input from the system coordinator and ε_x is the corresponding stop criterion.

6. Numerical test

In this section, the numerical behaviour of the SCM is investigated through an analytic test problem. Results are compared with those obtained via the ATC approach. The test problem is Golinski's speed reducer problem [19]. The reason for selecting this problem is that each subsystem involves only local and global shared variables, and the global objective is a function of the objectives of each subsystem, which are the same as the formulation of product family problem defined in Eq. (1).

According to the design problem defined in [19], the weight, or local objective, of the gear subsystem is $f_1 = F_1$, and its local constraints are $\boldsymbol{g}_1 = \boldsymbol{g}_{\text{gear}}$. The weights for shaft 1 and 2 subsystem are $f_2 = F_2 + F_4 + F_6$ and $f_3 = F_3 + F_5 + F_7$, respectively. Similarly, the local constraints are $\boldsymbol{g}_2 = \boldsymbol{g}_{\text{shaft, 1}}$ for shaft 1 and $\boldsymbol{g}_3 = \boldsymbol{g}_{\text{shaft, 2}}$ for shaft 2. The optimal result of the original all-in-one problem is $F(\boldsymbol{z})$=2994 (rounded) [20].

6.1. Solve via ATC

The original design problem is decomposed into a two-level ATC formulation with three subproblems. The problem decomposition is shown below:

$$\text{System Problem:} \quad \begin{aligned} &\text{find} \quad \boldsymbol{x}_s = [x_1, x_2, x_3]^T \\ &\min \quad f = F + \pi(\boldsymbol{c}) \\ &\text{s.t.} \quad \text{No Constraints} \\ &\text{where} \quad \boldsymbol{c} = [\boldsymbol{x}_s;\ \boldsymbol{x}_s;\ \boldsymbol{x}_s] - [\boldsymbol{x}_{s1};\ \boldsymbol{x}_{s2};\ \boldsymbol{x}_{s3}] \end{aligned} \qquad \text{Subproblem 1:} \quad \begin{aligned} &\text{find} \quad \boldsymbol{x}_{s1} = [x_1^{(1)}, x_2^{(1)}, x_3^{(1)}]^T \\ &\min \quad f_1 = F_1 + \pi(\boldsymbol{c}) \\ &\text{s.t.} \quad \boldsymbol{g}_1 = \boldsymbol{g}_{\text{gear}} \le \boldsymbol{0} \\ &\text{where} \quad \boldsymbol{c} = \boldsymbol{x}_s - \boldsymbol{x}_{s1} \end{aligned} \tag{6}$$

where the bracketed top-right index denotes the subsystem at which the shared variable copy is computed, and $\pi(\boldsymbol{c})$ is the penalty function with a quadratic form, that is $\pi(\boldsymbol{c}) = \left\| \boldsymbol{w} \circ \boldsymbol{c} \right\|_2^2$. The formulation of subproblem 2 and

3 are similar, thus the vector of shared variables is $\boldsymbol{x}_s = [x_1, x_2, x_3]^T$. Termination tolerances are set to $\varepsilon_F = 0.03\%$ and $\varepsilon_x = 0.01$. For all experiments, initial penalty parameters are $\boldsymbol{w}^{(1)} = \boldsymbol{0.1}$ and $\beta = 2$.

6.2. Solve via SCM

SCM uses the same problem decomposition strategy, yielding a system level problem and three subproblems. It is not difficult to find that the formulations of the three subproblems are exactly the same as the ATC method, while only the system level differs. This can be helpful for us to compare the efficiency of the system levels. The termination tolerances and initial penalty parameters are the same as that in the ATC method.

Convergence is achieved for the same four different starting points, which are shown in the Table 1. It can be seen that for the SCM, the convergence requires an average of 1226.5 function evaluations, which highlights an average computational saving about 25% over the ATC method. In addition, all the objective values converge to the global optimum, which are better and more stable compared with ATC approach. In summary, SCM robustly and efficiently solves this test problem.

Table 1: Solutions via ATC and SCM

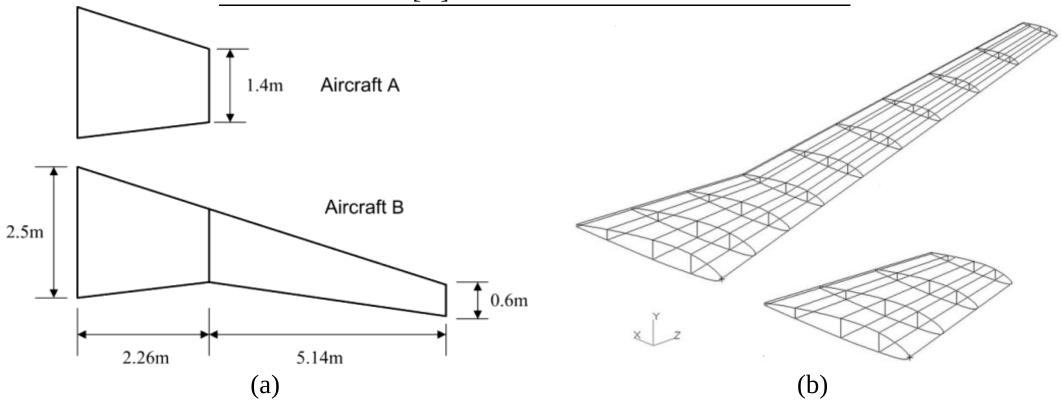
Starting Point	Optimum Objective		System Iterations		Function Evaluations	
	ATC	SCM	ATC	SCM	ATC	SCM
$\boldsymbol{z}$=[3.10,0.75,21.50,7.80,7.80,3.40,5.25]^T	2996	2994	25	17	1638	1268
$\boldsymbol{z}$=[2.60,0.70,17.00,7.30,7.30,2.90,5.00]^T	2995	2994	25	17	1636	1192
$\boldsymbol{z}$=[3.60,0.80,28.00,8.30,8.30,3.90,5.50]^T	2996	2994	25	17	1644	1253
$\boldsymbol{z}$=[3.60,0.80,17.00,7.30,8.30,2.90,5.00]^T	2995	2994	25	17	1623	1193

7. Wing structures family

The second problem describes the approach for optimization of wing structures for an unmanned aircraft family with reconfigurable wing component, which is more complicated and related to practical implementation issues. The aircraft family consists of two variants, A and B, with different aspect ratio. Aircraft A mainly performs the dash mission with a low aspect ratio wing, while aircraft B mainly performs the surveillance mission with a high aspect ratio wing. The wing structure of aircraft A is adopted by aircraft B as inner wing section, which means it is a shared component in the aircraft family. The mission requirements and reference wing properties are summarized in Table 2 and Figure 2(a).

Table 2: Aircraft design parameters

Parameter	Aircraft A	Aircraft B
Maximum takeoff weight [kg]	3000	3000
Cruise altitude [m]	7000	12000
Cruise speed [Mach]	0.8	0.23
Wing span [m]	6.52	16.8
Leading edge sweep [deg.]	20	20
Aspect ratio	4	12
Max load factor [G]	12	2.5



(a) (b)

Figure 2: Planform and finite element models of the two aircraft wings

The wing structures are modeled consisting of spars, ribs, and skin panels, as showed in Figure 2(b). The skin panels, spar webs and the ribs are modeled as shear panels, and the spar caps as rod elements. All these elements can be designed, whereas the leading edge, trailing edge and ribs are not designed. The goal of the wing structural optimization problem is to minimize the weight of the wing structure subject to stress and deflection constraints. The shape function linking method is adopted to reduce the number of design variables, and the value of linear function at a certain location determines the value of the property of that specific element. To simplify the computation, the span-wise distribution of the air load is based on Schrenk load distribution.

The upper and lower skin of wing box is subdivided into 2 regions by the spars, respectively. As the wing A is shared as the inner wing section of wing B, the shape function of wing B uses two independent linear functions to determine the size of each component, thus each component's shape function is determined by the property of elements at four locations, which means that there are 4 design variables for each component. As we can see from Table 3, there are 40 design variables in total for wing B, in which the Location 1 refers to wing root, Location 2 and 3 refer to the separation surface, and Location 4 located at wing tip. While for wing A, only the design variables at Location 1 and 2 are included, that is 20 variables in total. The individual optimal design problem for wing structure A and B is formulated as:

$$\text{A:}\quad \begin{array}{ll} \text{find} & \boldsymbol{x}_A \\ \text{min} & m_A(\boldsymbol{x}_A) \\ \text{s.t.} & \sigma_{\max} \le 222 \ \text{MPa} \\ & \sigma_{\min} \ge -167 \ \text{MPa} \\ & \delta \le 226 \ \text{mm} \end{array} \qquad \text{B:}\quad \begin{array}{ll} \text{find} & \boldsymbol{x}_B \\ \text{min} & m_B(\boldsymbol{x}_B) \\ \text{s.t.} & \sigma_{\max} \le 267 \ \text{MPa} \\ & \sigma_{\min} \ge -200 \ \text{MPa} \\ & \delta \le 840 \ \text{mm} \end{array} \tag{7}$$

where m is the weight of wing, and $\boldsymbol{x}_A = [x_{1A}, \cdots, x_{20A}]^T$ and $\boldsymbol{x}_B = [x_{1B}, \cdots, x_{40B}]^T$. $\sigma_{\max}$ and $\sigma_{\min}$ are the maximum tensile and minimum compression stresses, respectively. δ is the maximum deflection at wing tip.

It is assumed that the structures are manufactured from aluminum with a density of 2.7 g/cm^3 and an elasticity modulus of 7.0×10^4 MPa. The optima of individual design for wing A and B are 43.89 kg and 92.28 kg, respectively. The objective for the wing structures family is defined as the sum of normalized weight, that is:

$$F = m_A(\boldsymbol{x}_A)/43.89 + m_B(\boldsymbol{x}_B)/92.28 \tag{8}$$

As mentioned above, the vector of shared variables can be denoted as $\boldsymbol{x}_s = [x_{1A} \cdots x_{20A}]^T = [x_{1B} \cdots x_{20B}]^T$. Wing A has no individual variables, and the individual variables for wing B are $\boldsymbol{x}_{IB} = [x_{20B} \cdots x_{40B}]^T$.

Implement the proposed methodology to solve this design problem. The optimization converged after 11 iterations with a value of 2.219. The design results are presented in Table 3.

Table 3: Wing structures design results of aircraft family

Components	Location & Variable Values					
	Wing Structure A		Wing Structure B			
	1	2	1	2	3	4
Front spar [mm^2]	171.68	135.40	171.68	135.40	148.50	50.00
Internal spar [mm^2]	198.77	122.69	198.77	122.69	121.37	50.00
Rear spar [mm^2]	71.79	56.57	71.79	56.57	50.00	50.00
Front upper skin [mm]	3.84	2.51	3.84	2.51	2.79	0.55
Front lower skin [mm]	2.37	2.44	2.37	2.44	3.00	0.50
Rear upper skin [mm]	3.10	1.75	3.1	1.75	2.30	0.50
Rear lower skin [mm]	2.06	0.83	2.06	0.83	1.59	0.50
Front web [mm]	1.92	1.09	1.92	1.09	1.00	1.00
Internal web [mm]	1.22	2.00	1.22	2.00	1.00	1.00
Rear web [mm]	1.77	1.08	1.77	1.08	1.00	1.00
Weight [kg]	51.66		96.13			

It can be seen that both wing A and B pay a price due to the commonality decision of sharing components. Compared with the individual design results, the increased weight for two wing structures are 7.77 kg and 3.85 kg, respectively. The weight of wing A increased more than that of wing B, this is due to the fact that the wing tip area for aircraft A needs to support the load on the outer wing section when it is used as inner wing section on aircraft B. In addition, for wing B, it is important to note that the designed property on the outer wing section is larger than the values on inner wing section for part of components. The reason is that the inner wing section is shared by the two family members, to satisfy the deflection constraint at wing tip for wing B, increasing the stiffness of outer wing section leads to less increment of design objective F than that of inner section.

8. Conclusions

This article discusses the bi-level decomposition approach for the optimization of product families with

predefined platforms. For decomposition-based method, a system coordinator with global sense will greatly improve the optimization efficiency. Based on this consideration, a new SCM is proposed is in this article.
The innovation of the new method is the system coordinator, which not only provides consistent shared variables to subsystems, but also does the tradeoff between the all subsystems by the use of sensitivity information. The shared variables are determined by minimizing the total performance deviation with respect to the optimal design of each subproblems and constraints violation incurred by sharing. The first order Taylor series approximation is introduced to evaluate the values of performance deviation and constraints violation.
As many other decomposition-based methods, the family design problem is decomposed naturally by individual product. This decomposition by product variant provides many benefits, such as simplifying the analysis integration, reducing problem complexity, and enabling concurrent design of all product variants.
Results from the numerical test problem suggest that SCM can robustly and efficiently solve the test problem, which performs better than the ATC method. In addition, both the formulations of SCM and numerical test suggest that the SCM is also suitable to the MDO problems that the subsystems are only linked through a number of shared variables. This article also illustrates the successful application of SCM to the structural optimization problem of wing structures for an unmanned aircraft family.

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