Welded cellular cylindrical shell – a new structural solution for the optimum design of a cantilever column

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1. Abstract

A cantilever column is loaded by compression and bending. The horizontal displacement of the column top as well as the outside diameter of the cylindrical shell are limited. The strengthening of the column is performed in the lower part of the column only.

Three structural versions of the column are optimized and compared to each other.

Firstly, the unstiffened circular shell is optimized. It is found that the required large thickness is unsuitable for fabrication.

Secondly, the stringer stiffened circular shell is optimized. The halved rolled UC section stringers are used only in the lower part of the column, the distance of the interruption of stiffeners is also optimized. It is found that the required shell thickness is unsuitable for fabrication.

Thirdly, a new structural version, the cellular shell is used. Cellular shells are constructed from two circular cylindrical shells and a grid of stiffeners welded between them. They have similar advantages than the cellular plates, namely they can produce a large stiffness with small structural height. Their smooth surface is suitable for corrosion protection and they are more aesthetic than the stringer stiffened shells.

The parts of the outer circular shell are welded to the stringers from outer side with longitudinal fillet welds. Halved circular hollow section (CHS) stringers enable the easy welding of the outer fillet welds.

The unknown variables to be optimized are as follows: thicknesses of the inner and outer shell, dimensions and number of the halved CHS stiffeners as well as the distance of the interruption of stiffeners.

The study shows a realistic case when the cellular shell can be used with smaller shell thicknesses and lower cost than the shell stiffened with outer side stringers.

The displacement constraint is so strict that the stress, shell buckling and beam-column buckling constraints are passive.

The cost function to be minimized contents the cost of material, welding and painting. The optimization is performed by a systematic search using a MathCAD algorithm.

2. Keywords:
welded shell structures, structural optimization, cost calculation, cellular structures, cantilever columns

3. Introduction

Similar to cellular plates [1, 2, 3] the cellular shells are constructed from two circular cylindrical shells and a grid of stiffeners welded between them (Fig. 3). It is advantageous to use halved circular hollow section (CHS) stiffeners, since the parts of the outer circular shell can easily be welded to them.

The aim of present study is to show the advantages of cellular shells over the stringer stiffened ones. Their large stiffness and small structural height can be useful for a compressed and bent cantilever column in the case of a strict constraint on horizontal displacement of the top together with a constraint on maximum diameter.

The study compares three structural versions for the welded circular cylindrical shell as follows: (a) unstiffened, (b) stiffened with halved rolled I section stiffeners, and (c) stiffened by cellular shell.

The basis of the comparison is the cost, which contents the cost of material, welding and painting.

The base of the column is built-up, but the structural solution of the foundation and its cost is not treated.

Given data: column height $L = 15$ m, factored compression force $N_F = 2 \times 10^7$ [N], horizontal force $H_F = 0.1 N_F$, yield stress of steel $f_y = 355$ MPa, elastic modulus $E = 2.1 \times 10^5$ MPa. In the calculation of displacement the horizontal force is divided by the safety factor $\gamma_M = 1.5$.

Constraints: limitation of the horizontal displacement of the column top: $w_{max} = L/\phi, \phi = 1000$ and limitation of the shell diameter: $D = 2R = 3000$ mm.

4. The unstiffened shell (Fig.1 without stiffeners)

It can be concluded that the solutions need very thick shell parts, not suitable for fabrication. It should be noted that the constraint on beam-column buckling is passive in this case.
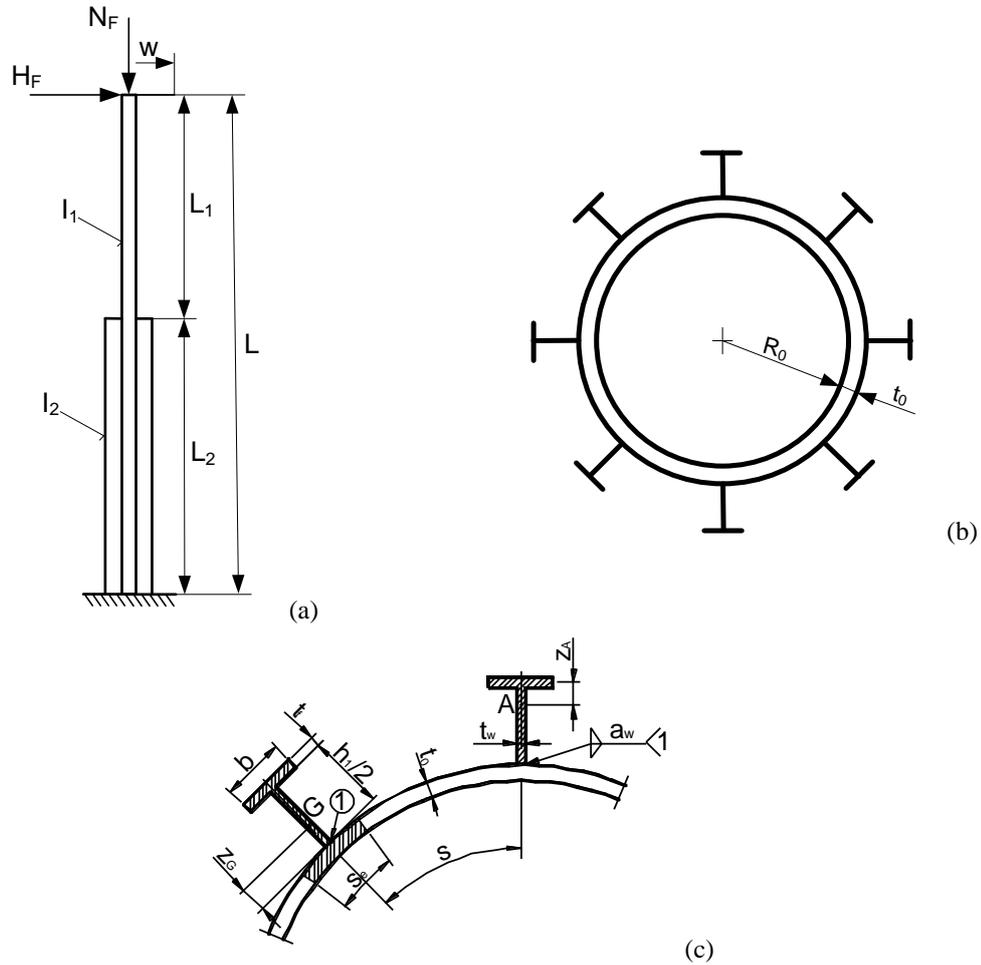

Figure 1: (a) Cantilever column loaded by a compressive and a horizontal force, the horizontal displacement w and the outside diameter are limited, (b) cross-section of the column, (c) dimensions of the stringer stiffeners

5. The shell stiffened with halved rolled I section stiffeners from outside (Fig.1)

Table 1: Some results of the optimization. Dimensions in mm, volume in mm^3 and costs in \$. The optima are marked by bolt letters

h	222.2	222.2	161.8	152.4
b	209.1	208.1	154.4	152.2
t_w	12.7	12.7	8	5.8
t_f	20.5	20.5	11.5	6.8
t	45	46	42	42
n_s	18	18	20	20
L_2	12000	9000	9500	9000
w	14.99	14.99	14.84	14.96
$10^{-9}V$	7.0130	6.847	6.666	**6.545**
K	96480	94850	88500	**88010**

The constraints on panel buckling and beam-column buckling (see section 3) are passive. It can be seen that the decrease of L_2 and the dimensions of stiffeners gives less volume and cost.

The main problem is the large shell thickness (over 40 mm), which is unsuitable for fabrication.

6. The column stiffened by cellular shell (Figs. 2, 3)

Halved circular hollow section (CHS) [4] stiffeners are used. This type of stiffeners have more advantages as follows: (a) they enable suitable welded joints for the cover shell elements, (b) their large torsional stiffness gives a large overall stiffness for the whole structure.

6.1 Geometric characteristics

The cross-sectional area of a half CHS is (Fig. 2)

$$A_s = \pi R_s t_s \ , \ R_s = \frac{D_s - t_s}{2} \tag{1}$$

the distance of its gravity centre is

$$y_G = \frac{2R_s}{\pi} \tag{1a}$$

and its moment of inertia

$$I_s = \frac{R_s^3 \pi}{2}\left(1 - \frac{8}{\pi^2}\right) \tag{2}$$

The distance of its gravity centre to the shell centre

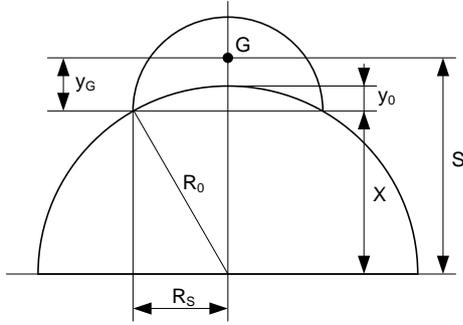$$s = y_G + X, X = \sqrt{R_0^2 - R_s^2}, y_0 = R_0 - X \tag{3}$$

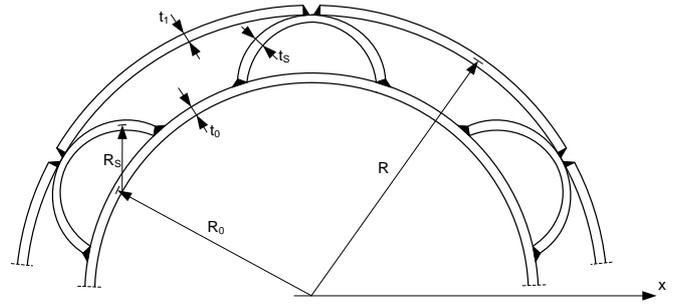Figure 2: Geometry of the cellular shell   Figure 3: Dimensions of a cellular shell

The radius of the inner shell R_0 can be calculated from the following equation

$$R = s + \frac{t_s + t_1}{2} \tag{4}$$

(t_1 is the thickness of the outer shell)

$$R_0 = \sqrt{\left(R - \frac{2R_s}{\pi} - R_s - \frac{t_s + t_1}{2}\right)^2 + R_s^2} - \frac{t_0}{2} \tag{5}$$

The moment of inertia of n_s stiffeners about the centre of the shell

$$I_{xs} = \left(I_s + A_s s^2\right)\sum_{i=1}^{n_s} \cos^2\left(\frac{2\pi i}{n_s}\right) + \left(\frac{\pi R_s^3 t_s}{2}\right)\sum_{i=1}^{n_s} \sin^2\left(\frac{2\pi i}{n_s}\right) \tag{6}$$

The moment of inertia of the whole cellular shell (Fig.3)

$$I_x = \pi R_0^3 t_0 + \pi R^3 t_1 + I_{xs} \tag{7}$$

Fabrication constraint to enable the welding of the half CHS to the inner shell:
from

$$\frac{2R_0 \pi}{n_s} - 2R_s \geq 2t_s \tag{8}$$

the maximum allowable number of half CHS stiffeners

$$n_{s.\max} = \frac{\pi R_0}{R_s + t_s} \tag{9}$$

6.2 Constraint on horizontal displacement of the column top

$$w_{max} = \frac{H_F}{EI_1\gamma_M}\left[\frac{(L-L_2)^3}{3} + \mu\left(L-\frac{L_2}{2}\right)(L-L_2)L_2 + \mu\frac{L_2^2}{2}\left(L-\frac{L_2}{2}\right)\right] \le \frac{L}{1000} \tag{10}$$

where

$$\mu = \frac{I_1}{I_2}, \gamma_M = 1.5, \quad I_1 = \pi R_0^3 t_1, I_2 = I_x \tag{11}$$

I_i and I_2 are the moments of inertia of the upper and lower column part, respectively, γ_M is the safety factor.

6.3 Constraint on panel shell buckling of the outer shell parts between stiffeners
According to the Det Norske Veritas [5] design rules for shell buckling

$$\sigma_{max} = \frac{N_F}{A} + \frac{H_F L}{W_x} \le f_{ks} = \frac{f_y}{\sqrt{1+\bar{\lambda}^4}}, \quad \bar{\lambda} = \sqrt{\frac{f_y}{f_E}} \tag{12}$$

$$f_E = C\frac{\pi^2 E}{12(1-v^2)}\left(\frac{t_1}{s_0}\right)^2, C = \psi\sqrt{1+\left(\frac{\rho_0\xi}{\psi}\right)^2} \tag{13}$$

$$\psi = 4, \rho_0 = 0.5\left(1+\frac{R}{150t_1}\right)^{-0.5}, \xi = 0.702Z \tag{14}$$

$$Z = \frac{s_0^2}{Rt_1}\sqrt{1-v^2}, v = 0.3 \tag{15}$$

In the case of such a very strict displacement constraint the panel buckling constraint is not active. σ_{max} is so small that the effective shell width is equal to the whole width s_0.
Calculations show that this constraint is not active.

6.4 Constraint on beam-column buckling
The check should be performed by taking into account the overall buckling of the column [6]. For the calculation of the Euler critical stress the formula given by Timoshenko and Gere [7] is used.

$$\frac{N_F}{\chi_1 A f_y} + k_{yy1}\frac{H_F L}{W_x f_y} \le 1 \tag{16}$$

$$\chi_1 = \frac{\phi_1 - \sqrt{\phi_1^2 - \lambda^2}}{2\lambda^2}, \quad \phi_1 = 1 + \alpha(\lambda - 0.2) + \lambda^2, \quad \lambda = \sqrt{\frac{f_y}{\sigma_E}}, \quad \sigma_E = \frac{\pi^2 EI_2}{4L^2 AC} \tag{17}$$

$$C = \frac{L_2}{L} + \frac{L_1 I_2}{L I_1} - \frac{1}{\pi}\left(\frac{I_2}{I_1} - 1\right)\sin\frac{\pi L_2}{L} \tag{18}$$

$$\alpha = 0.34,$$

$$k_{yy1} = C\cdot_{my}\left(1+0.6\lambda\frac{F}{\chi_1 A f_y}\right) \quad \text{for} \quad \lambda < 1 \tag{19a}$$

$$k_{yy1} = C\cdot_{my}\left(1+0.6\frac{F}{\chi_1 A f_y}\right) \quad \text{for} \quad \lambda \ge 1 \tag{19b}$$

$$C_{my} = 0.6$$

6.5 The cost function

The cost of welding is formulated according to the fabrication sequence [8 - 11].

(1) Fabrication of 5 shell elements of length 3 m without stiffeners. For one shell element 2 axial butt welds are needed (GMAW-C) (K_{F1}). The cost of forming of a shell element into the cylindrical shape is also included (K_{F0}).

(2) Welding of the whole unstiffened shell from 5 elements with 4 circumferential butt welds (K_{F2}). Θ is the factor expressing the complexity of assembly

$$V_1 = 3000x2R_0\pi t_0, \rho = 7.86x10^{-6}\,\text{kgmm}^{-3}, k_F = 1.0\,\text{\$/min}, k_{M1} = 1.0\,\text{\$/kg}. \tag{20}$$

$$K_{F0} = k_F\Theta e^{\mu}, \mu = 6.8582513 - 4.527217t_0^{-0.5} + 0.009541996(2R_0)^{0.5} \tag{21}$$

$$K_{F1} = k_F\left[\Theta\sqrt{\kappa\rho V_1} + 1.3x0.152x10^{-3}t_0^{1.9358}(6000 + 2\pi R_0)\right] \tag{22}$$

$$\kappa = 4, V_1 = 2R_0\pi t_0 x3000, \Theta = 2 \tag{23}$$

$$K_{F2} = k_F\left(\Theta\sqrt{25\rho V_1} + 1.3x0.152x10^{-3}t_0^{1.9358}x4x2R_0\pi\right) \tag{24}$$

(3) Welding of the half CHS stiffeners to the base shell using SAW fillet welds

$$K_{F3} = k_F\left[\Theta\sqrt{(n_s + 1)\rho V_2} + 1.3x0.2349x10^{-3}a_w^2 2L_2 n_s\right] \tag{25}$$

$$V_2 = 5V_1 + n_s A_s L_2, \quad a_w = 0.3t_s \quad (a_{min} = 3\,\text{mm}) \tag{26}$$

(4) Forming of the outer curved shell panels of length 3 m

$$K_{F4} = k_F\Theta e^{\mu_1}, \quad \mu_1 = 6.8582513 - 4.527217t_1^{-0.5} + 0.009541996(2R)^{0.5} \tag{27}$$

(5) Welding of an outer curved shell panel of length L_2 using κ_3 shell parts of length 3 m by (κ_{30}-1) GMAW-C butt welds

$$K_{F5} = k_F\left(\Theta\sqrt{\kappa_{30}\rho V_3} + 1.3x0.152x10^{-3}t_1^{1.9358}\frac{2R\pi}{n_s}(\kappa_{30} - 1)\right) \tag{28}$$

$$V_3 = \frac{2R\pi t_1 L_2}{n_s} \tag{29}$$

$$\kappa_3 = \frac{L_2}{3000} \text{ round up to } \kappa_{30}$$

(6) Welding of the outer panels to the stiffened shell by SAW fillet welds of size $a_{w1} = 0.3t_s$

$$K_{F6} = k_F\left(\Theta\sqrt{(n_s + 1)\rho V_4} + 1.3x0.2349x10^{-3}a_{w1}^2 2n_s L_2\right) \tag{30}$$

$$V_4 = V_2 + n_s V_3 \tag{31}$$

Painting cost

$$K_P = k_p S, S = 2\pi R_0(2L - L_2) + 2R\pi L_2 \tag{32}$$

Material cost

$$K_M = k_M\rho V_4 \tag{33}$$

The total cost

$$K = K_M + 5(K_{F0} + K_{F1}) + K_{F2} + K_{F3} + \kappa_{30}K_{F4} + n_s K_{F5} + K_{F6} + K_P \tag{34}$$

The details of the search are shown in Table 2. For fabrication aspects the following limits are introduced: $t_{0max} = 30$, $t_{1max} = 30$, $t_{smin} = 10$ mm.

Table 2: Details of the optimization. The maximal displacement in each case is near the allowable value of 15 mm. Dimensions in mm, volume V in mm^3. The optimum is marked by bold letters

t_0	t_1	L_2	$V_4 \times 10^{-9}$	K ($)
25	29	8900	5.768	78460
24	30	8900	5.720	**77440**
23	30	9300	**5.705**	79410
22	30	9800	5.719	78740
21	30	10300	5.733	78100
20	30	10900	5.776	77740
19	30	11700	5.876	77940
18	30	13200	6.1780	82920

The numerical values show the following results:
(a) The minimum volume and cost is found for the minimum number of stiffeners $n_s = 4$:
(b) The CHS stiffener profile of 101.6x10 gives the minimum volume and cost. The tendency is to minimize D_s to maximize R_0, and to maximize t_s, thus, we select this profile.

7. Conclusions

A realistic numerical problem is investigated, in which the outer shell diameter and the horizontal displacement of the column top is limited. The numerical value of the compression force N_F is also given. Three structural solutions are optimized:
(1) the unstiffened circular cylindrical shell has the minimal volume $V = 5.316s \times 10^9$ mm^3 but the shell thickness is 50 mm, which is unsuitable for fabrication,
(2) the circular shell stiffened with halved rolled UC sections has the structural volume $V = 6.545 \times 10^9$ mm^3 and the cost $K = 88010$ $, but the shell thickness is 42 mm, unsuitable for fabrication,
(3) the optimum solution of the column strengthened by cellular shell has values $V = 5.705 \times 10^9$ mm^3 and $K = 77440$ $, thickness 30 mm.
It can be concluded that in this case only the cellular shell version can fulfil all the requirements (horizontal displacement, maximum outer diameter, maximum thickness of 30 mm, minimum cost). In addition, the cellular shell can be more easily protected against corrosion and is much more aesthetic than the second version.

8. Acknowledgements

9. References

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