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A level set method for the representation of multiple types of boundaries and its application in structural shape and topology optimization

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1. Abstract

A level set method is developed for the representation of multiple types of boundaries, and it is applied in two structural shape and topology optimization problems. The first problem is the optimization of both structure and support, where both the homogeneous Neumann boundary and the Dirichlet boundary are optimized. The second problem is the optimization of structures subjected to pressure load, where both the non-homogeneous and homogeneous Neumann boundaries are optimized. In order to address the issue that how to represent different types of boundaries of a structure, a new scheme of representation is proposed. Two independent level set functions are used for the representation. The two types of boundaries are represented separately and are allowed to be continuously propagated during the optimization. The optimization problem of minimum compliance is considered.

2. Keywords: topology optimization, level set method, support, pressure load.

3. Introduction

A structure has several types of boundaries, for instance the Dirichlet boundary, the Neumann boundary, and the free boundary. Optimization of any one of them will be helpful to improve the performance of a structure, and there are engineering applications that require to simultaneously optimize two or more boundaries of a structure. For instance, in the optimization of structure with pressure load, both the free boundary and the Neumann boundary are required to be simultaneously optimized [1–11]. Also, it would be better if both the free boundary and the Dirichlet boundary (support) are simultaneously optimized [12–18].

In conventional level set method of structural shape and topology optimization [10, 19–22], usually only the traction free boundary of a structure is represented through a single level set function and is optimized, but the Dirichlet boundary and the Neumann boundary are specified before and fixed during the optimization. If multiple types of structure boundaries are to be simultaneously optimized, it seems that a natural extension of the conventional level set method is to use the zero level set of a single level set function to represent the multiple types of boundaries, and then impose different labels on the points of these boundaries, and track these points during the optimization. However, it is well known that such a tracking method is not able to deal with topological changes [10, 19–22]. Therefore, the conventional representation of a structure through a single level set function is not adequate for the simultaneous optimization of multiple types of boundaries.

Such a situation leads to a fundamental issue of the present study, i.e., how to represent multiple types of boundaries of a structure. In order to address this issue, a new scheme of representation through two level set functions is proposed, and it is applied in two structural shape and topology optimization problems.

4. Representation of structure and multiple types of boundaries

The scheme of representation is shown in Fig. 1. Two independent level set functions, denoted by Φ and Ψ, are used to represent a structure. The inside and outside regions with respect to the structure boundary are given by

$$\Omega = \{x \,|\, \Phi(x) < 0 \text{ and } \Psi(x) < 0,\; x \in \mathscr{D}\} \tag{1}$$

$$\mathscr{D} \backslash \overline{\Omega} = \{x \,|\, \Phi(x) > 0 \text{ or } \Psi(x) > 0,\; x \in \mathscr{D}\} \tag{2}$$

where $\mathscr{D}$ is the reference domain. In other words, a structure is represented by the set of points where both of the two level set functions are negative. The level set function Φ or Ψ is constructed to be a signed distance function to a closed curve in 2D or a closed surface in 3D, and a rectilinear grid is used in the numerical implementation. With such a representation, the structure defined in Eq. (1) can be obtained through a boolean operation of the two level set functions

$$\Omega = \{x \,|\, \max\{\Phi(x), \Psi(x)\} < 0,\; x \in \mathscr{D}\} \tag{3}$$

In addition, many geometric properties including the unit outward normal n and the curvature κ can be readily expressed [23].

The free boundary to be optimized is represented by a portion of the zero-level set of Φ, i.e.,

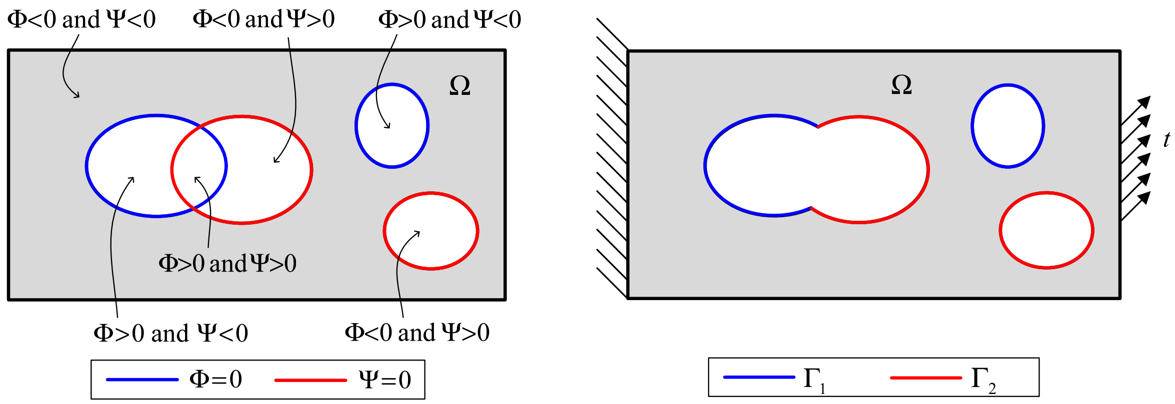
$$\Gamma_H^\Phi = \{x \,|\, \Phi(x) = 0, \Psi(x) < 0,\; x \in \mathscr{D}\} \tag{4}$$

In the shape and topology optimization of both structure and support, the Dirichlet boundary to be optimized is represented by a portion of the zero-level set of Ψ, i.e.,

$$\Gamma_D = \{x \,|\, \Psi(x) = 0, \Phi(x) < 0,\; x \in \mathscr{D}\} \tag{5}$$

Similarly, in the shape and topology optimization of structures subjected to pressure load, the Neumann boundary Γ_N to be optimized is represented by a portion of the zero-level set of Ψ, i.e.,

$$\Gamma_N = \{x \,|\, \Psi(x) = 0, \Phi(x) < 0,\; x \in \mathscr{D}\} \tag{6}$$



(a) representation of shape (b) representation of boundary

Figure 1: The scheme for the representation of shape and boundary.

During the optimization, the two types of boundaries are independently and continuously propagated. Propagation of the two boundaries is modeled separately by two independent Hamilton–Jacobi equations [10, 19–22]

$$\Phi_t + F^\Phi \cdot \nabla\Phi = b^\Phi \kappa |\nabla\Phi| \tag{7}$$

$$\Psi_t + F^\Psi \cdot \nabla\Psi = b^\Psi \kappa |\nabla\Psi| \tag{8}$$

where the velocity termsF^Φ, b^Φ, F^Ψ, b^Ψ are given by the shape derivatives of the optimization problem.

The concept of using multiple level set functions in the structural shape and topology optimization had been introduced in the design of structures that have several different materials [24, 25]. The idea is to use n level set functions to represent up to 2n different materials. In the present study, two level set functions are not used to represent multiple regions of different materials, but they are used to represent two different types of boundaries.

5. Shape and topology optimization of both structure and support

The effects of the support on a structure's performance have been considered in optimization for a long time. In the early pioneering research work [26–30], the optimal position and stiffness of supports of discrete structures were considered. Recently, the optimization of support was extended for continuum structures. In these studies, the methods of optimization are divided into two categories. The first category is based on a field of background springs that represent potential supports [12–14]. A continuous design variable is given to each spring. When the value of a spring variable reaches the lower bound, the point where the spring is attached to the structure is considered as free. On the other hand, when the value of a spring variable reaches the upper bound, the attachment point is considered as fixed. Intermediate values of spring variables are penalized. The second category is based on continuous variation of support point or support boundary [15–18], which is similar to the shape optimization.

The optimization problem considered in the present study is the regularized minimum compliance problem

$$\inf_{\Omega \in \mathscr{U}_{\mathrm{ad}}} J(\Omega) + \ell P(\Omega) \tag{9}$$

where J is the compliance; $\ell > 0$ is a fixed weighting parameter; $P(\Omega)$ is the perimeter of a structure. The set of admissible shapes is defined as $\mathscr{U}_{\text{ad}} = \{\Omega \subset \mathscr{D},\, V(\Omega) \leq \overline{V},\, C(\Gamma_D) \leq \overline{C}\}$ where $V(\Omega) \leq \overline{V}$ is a volume constraint; $C(\Gamma_D) \leq \overline{C}$ is a cost constraint of support given as

$$C(\Gamma_D) = \int_{\Gamma_D} c(x)\, \mathrm{d}s \leq \overline{C} \tag{10}$$

where Γ_D is the Dirichlet boundary; $c(x)$ is a fixed scalar field that describes the cost of support at point x. The function $c(x)$ means that the cost of support depends on the position.

In the optimization, both the free boundary Γ_H and the Dirichlet boundary Γ_D are optimized. The results of shape derivative analysis are given by

$$\mathscr{L}'(\Omega)(\theta) = \int_{\Gamma_H} G_{\Gamma_H} \theta \cdot n \mathrm{d}s + \int_{\Gamma_D} G_{\Gamma_D} \theta \cdot n \mathrm{d}s \tag{11}$$

where $\mathscr{L}$ is the Lagrangian; G_{Γ_H} and G_{Γ_D} are given by

$$G_{\Gamma_H} = -Ae(u) \cdot e(u) + \ell\kappa + \ell_1 \tag{12}$$

$$G_{\Gamma_D} = Ae(u) \cdot e(u) + \ell\kappa + \ell_1 + \ell_2 \left(\frac{\partial c}{\partial n} + \kappa c \right) \tag{13}$$

where f is the body force; u is the displacement; A is the elasticity tensor; $e(u)$ is the strain tensor; ℓ_1 and ℓ_2 are respectively the Lagrange multipliers for the volume constraint and the cost constraint of support; κ is the curvature of boundary.

The shape derivatives lead to the the velocity terms in Eq. (7) and (8) as

$$F^{\Phi} = (Ae(u) \cdot e(u) - \ell_1)n, \quad b^{\Phi} = \ell \tag{14}$$

$$F^{\Psi} = \left(-Ae(u) \cdot e(u) - \ell_1 - \ell_2 \frac{\partial c}{\partial n} \right) n, \quad b^{\Psi} = \ell + \ell_2 c \tag{15}$$

More details about the optimization problem and the shape derivative analysis are referred to [31].

6. Shape and topology optimization of structures subjected to pressure load

The problem of shape and topology optimization of structures subjected to pressure load is in most cases solved by using the SIMP method [32, 33]. Nevertheless, difficulties exist in the SIMP based solution, since one needs to find the pressure boundary from a smooth scalar field that represents the distribution of material in a reference domain. If the scalar field varies smoothly from 0 to 1 (unfortunately, this is true particularly in the early stage of optimization), the boundary of a structure and the pressure load is ambiguously defined. Several methods were proposed and integrated in the SIMP method to find the pressure boundary [1–5]. On the other hand, several creative approaches were proposed to mimic the effects of a pressure load by artificially incorporating another physical field into the optimization problem [6–9], hence circumventing the issue of pressure boundary. Nevertheless, the downside of these approaches is that some new numerical issues may arise [7, 8] and the optimization is more complex. The level set method was also used to solve the optimization with pressure load, for example the study by Allaire et al. [10] and by Guo et al. [11]. In these studies, the pressure boundary together with the free boundary are represented by the zero level set of a single level set function, then the pressure boundary is picked out from the zero level set by checking whether the normal vector is along a specified direction [10, 11]. Nevertheless, when the pressure load comes from several different directions, it will be much more complicated for these approaches to deal with the pressure boundary.

The optimization problem is also the regularized minimum compliance problem as given by Eq. (9), except that here is no constraint on the cost of Dirichlet boundary in the set of admissible shapes $\mathscr{U}_{\text{ad}}$, i.e., $\mathscr{U}_{\text{ad}} = \{\Omega \subset \mathscr{D},\, V(\Omega) \leq \overline{V}\}$.

In the optimization, both the free boundary Γ_H and the Neumann boundary Γ_N are optimized. The results of shape derivative analysis are give by

$$\mathscr{L}' = \int_{\Gamma_H} G_{\Gamma_H} \theta \cdot n \, \mathrm{d}s + \int_{\Gamma_N} G_{\Gamma_N} \theta \cdot n \, \mathrm{d}s \tag{16}$$

where $\mathscr{L}$ is the Lagrangian; G_{Γ_H} and G_{Γ_N} are given by

$$G_{\Gamma_H} = -Ae(u) \cdot e(u) + \ell\kappa + \ell_1 \tag{17}$$

$$G_{\Gamma_N} = -2\mathrm{div}(p_0 u) - Ae(u) \cdot e(u) + \ell + \ell_1 \tag{18}$$

where p_0 is the constant magnitude of a pressure load, and in the present study, the magnitude of pressure load is assumed to be a constant, i.e., $p = -p_0 n$; ℓ_1 is the Lagrange multiplier for the volume constraint.

The shape derivatives lead to tentative velocity terms in Eq. (7) and (8) as

$$F^{\Phi} = \left(Ae(u) \cdot e(u) - \ell_1\right) n, \quad b^{\Psi} = \ell \tag{19}$$

$$F^{\Psi} = \left(2\mathrm{div}(p_0 u) + Ae(u) \cdot e(u) - \ell_1\right) n, \quad b^{\Phi} = \ell \tag{20}$$

For most of the engineering applications considered in the literature, the pressure boundary Γ_N should not touch or cross the traction free boundary Γ_H. However, according to our experience of the present study, if the velocity terms are chosen according to Eqs. (19–20), the two boundaries may indeed touch or cross each other when the initial design is not so good or the size of descent steps of optimization is big. In order to address this issue, we modify the velocity vector F^{Ψ} on boundary Γ_N and modify F^{Φ} on boundary Γ_H, then we extend the modified velocities from the two boundaries to the entire reference domain by using a PDE based method [34]. More details about the shape derivative analysis and modification of velocity terms are referred to [35].

7. Numerical examples

7.1 Example 1: optimization of structure and support

The design problem is shown in Fig. 2(a). The shaded rectangular region at the middle is the admissible domain for the design of support. We do the finite element analysis by modifying a fixed background triangle mesh and do not use the artificial weak material, as proposed in our previous study [36]. The upper bound of volume is 0.35 m^3, and the upper bound of the cost of support is 0.3. The initial design is shown in Fig. 2(b).

The optimal structure is shown in Fig. 2(c). The objective function of the optimal structure is 10.93; compliance is 10.74; the volume is 0.35; the cost of support of the optimal design is 0.29. It can be seen that the optimal support does not coincide with the two dash lines. In addition, it is interesting to see that in the design shown in Fig. 2(c) the three loads are connected and that the two loads at the bottom are effectively used to counteract the upper load, thus minimizing the compliance due to these loads. From this example, we see that for a structure that is subject to multiple loads, load path can be tailored by the support such that the loads are self-equilibrated. In such cases, the design of support is important.

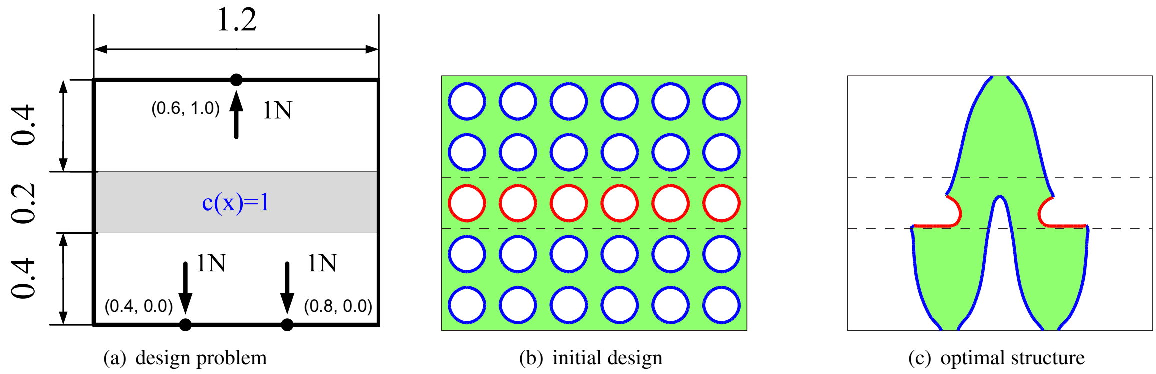


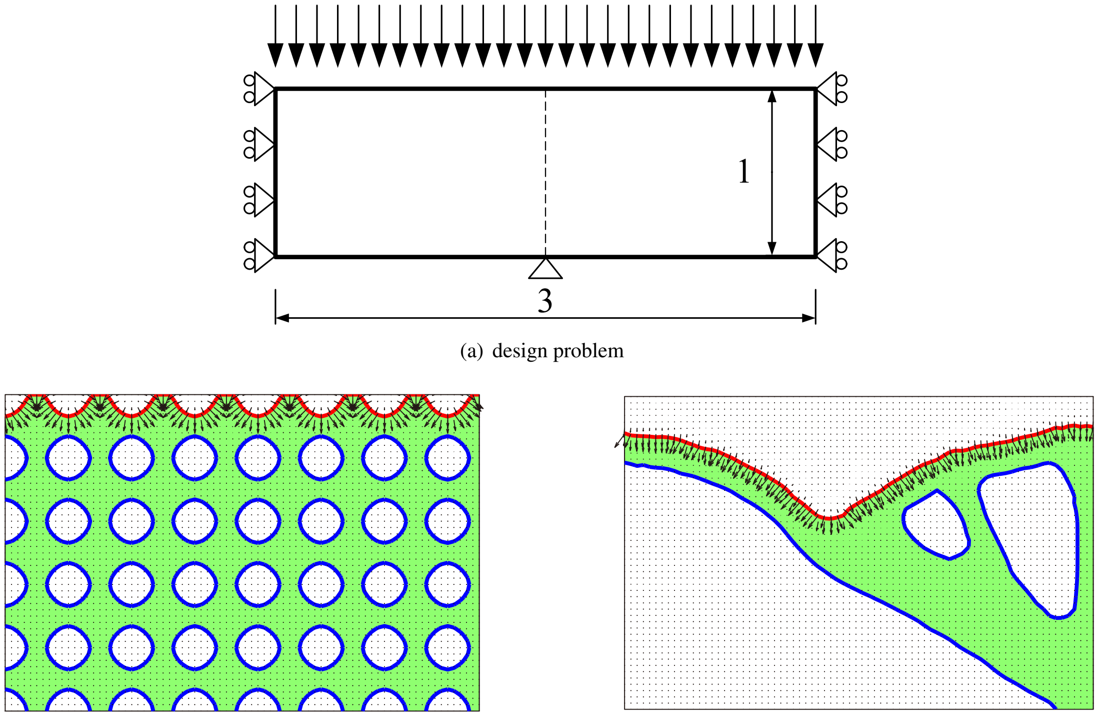
Figure 2: Design problem, initial design, and optimal structure.

7.2 Example 2: optimization of structure subject to pressure load

The optimal design problem of the third example is shown in Fig. 3(a). The pressure load is applied from the top of the structure. The reference domain is a rectangle of size 3 m $\times$ 1 m. The upper bound of volume is 0.9 m^3. An Eulerian method employing a fixed mesh and ersatz material [10] is used for the finite element analysis. The pressure load is converted to a volume force as proposed in [10]. Only the left half is analyzed in this example. The initial design is shown in Fig. 3(b). The optimal structure is shown in Fig. 3(c).

8. Conclusions

A new scheme of representation is proposed to represent multiple types of boundaries of a structure. Two independent level set functions are used for the representation. The two types of boundaries are represented separately and are allowed to be continuously propagated during the optimization. The optimization problem of minimum compliance is considered. The numerical examples demonstrated that the proposed method is effective. In the future,



(a) design problem

(b) initial design

(c) optimal design

Figure 3: Design problem, initial design, and optimal design.

the proposed method will be applied to many optimization problems that involve with multiple type of boundaries.

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