

## Continuous adjoint based topology optimization of a constrained thermal-fluid system

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### Abstract

This paper presents an approach for topology optimization of a coupled heat transfer and laminar fluid flow system. In this system, thermal gradient along the boundary of multiple heat sources needs to be controlled. The design objectives are to minimize the temperature of the domain, the fluid power dissipation and the tangential-thermal-gradient (TTG) along the boundary of the heat sources. The first two objectives are combined into a single cost function with weight variables. The point-wise TTG is aggregated via the Kreisselmeier–Steinhauser (KS) function into a constraint for the topology optimization. A gradient-based approach is developed to obtain the optimized designs. The finite element method is used to solve the coupled thermal-fluid equations with Taylor-hood elements for the Navier-Stokes equation and linear elements for the heat transfer equation. To ensure the robustness of the resulting design, a partial differential equation based density filter is used.

A salient feature of this approach is the use of the continuous adjoint method to derive gradients of the cost function and the TTG constraint with respect to the design variables. Because both the cost function and the TTG constraint involve the state variables in the governing equations, two sets of adjoint equations are thus needed, whereby each set of adjoint equations consist of an adjoint heat equation and an adjoint fluid equation. We derive analytical forms for the two sets of adjoint equations and boundary conditions.

Numerical examples are presented to demonstrate the effects of different weights in the thermal-fluid cost function on the optimized designs. A system energy balance and pumping power analysis provides further quantitative insight into the performance of the designs. The effects and benefits of the point-wise TTG constraint on the optimized designs are also illustrated. The optimized designs exhibit a clear solid-fluid boundary making them suitable for manufacturing.