Stabilisation of a Trapped Vortex for Enhancing Aerodynamic Flows

A. Bouferrouk\(^1\), S. I. Chernyshenko\(^1\)

\(^1\)AFM Research Group, School of Engineering Sciences
Southampton University, Southampton, SO17 1BJ, United Kingdom

Abstract
This study reports on the technologically important application of active stabilisation control of a trapped vortex concept for improving the aerodynamic characteristics of a Lighthill’s airfoil. The flow under consideration is two-dimensional, incompressible and inviscid solved using a suitably designed Discrete Vortex Method (DVM) code. With suction, various techniques were implemented for stabilisation resulting in reduced drag and increased lift. Stabilisation was also achieved via a feedback control law which kept the vortex stable with respect to large-scale vortex shedding.

Introduction
Large-scale vortex structures usually tend to travel downstream to the wake, and continuously maintain a chain of vortices forming behind them, see figure 1. Consequently, the drag increases and both the wake and the loads on the body become unsteady. However, using specific technologies such as vortex cells (cavities) allows the vortex structure to be held near the body at all times. This is what is known as a trapped vortex as depicted in figure 2. In figure 2 the trapped vortex is identified by the enclosed organised streamlines. Although not a new idea, only few articles are available on trapped vortices. Unfortunately, compared to other pursued ideas, it is much less known within the academic community. Research into trapped vortex flows began following Kasper’s seminal efforts in designing a glider having a significant lift improvement at low speeds (without corresponding change in drag), which he attributed to a massive vortex residing over the upper surface of the wing [6]. A trapped vortex is also a useful tool for drag reduction because it could postpone/prevent vortex shedding from bodies of aircraft, racing cars, and helicopters. Also, by maintaining a vortex near a wing’s surface it is possible to extend the post-stall performance by increasing the maximum obtainable lift. For a racing car, this translates into increased downforce as found by Garcia et al. [9]. Bunyakin et al. [5] considered trapped vortices for separation control which is extremely important in internal systems such as a compressor. There is also the opportunity of using a trapped vortex for vehicle control.

Unfortunately, previous studies concluded that a trapped vortex is unstable, or at best has a very limited region of stability [11], and as such the slightest form of perturbation may cause it to become unstable. Although a trapped vortex minimises unsteadiness, other forms of unpredictable perturbations and/or intrinsic instability mechanisms are always present in a flow making the vortex susceptible to being easily displaced. Therefore, to fully achieve the potential benefits of a trapped vortex, it must be stable at all times.

The intention of the ongoing work is to demonstrate the feasibility of stabilising a trapped vortex using active flow control. The main objective is to maximise the benefits of stabilisation by exploring reliable and cost effective techniques for enhancing aerodynamic flows.

Novelty of the Research Study
In past numerical work the modelling and stability of a trapped vortex was always based on a single point vortex. In this study, the vortex is modelled by a very large number of elemental blob vortices due to a DVM. However, use of single vortices enabled the application of linear stability theory for characterising the stability of the vortex while this is extremely difficult with a large number of vortices. Therefore, characterisation of stability will be different to old methods but not without proper physical justification. With the exception of the work of Iollo et al. [7] on optimal control of a single point trapped vortex, stabilisation has always been passive. The present work is a unique application of feedback control to the stabilisation of a trapped vortex.

Formulation of the Problem
The flow model for the study is the two-dimensional, inviscid, incompressible flow, while the simulation model for a body with a trapped vortex is a Lighthill’s airfoil. The body shape was determined from a classical inverse problem. The inverse problem is that of determining the body shape for a given velocity distribution on its surface. For the present geometry, the desired velocity distribution ensures no separation takes place except at a single point on the upper surface. Such point could be replaced by a cavity for vortex trapping [4], see figure 3. Thus, the Lighthill’s airfoil has a naturally desirable distribution of velocity and is potentially able to trap a large vortex. Because of these characteristics of the Lighthill’s airfoil it was selected as the simulation model. The flow solution was obtained using the inviscid version of a DVM due to Spalart [12]. Figure 4 shows a convergence study for the main parameters. The set of results suggests that a choice of 1300 vortices and a time step of 0.004 should provide reasonable estimates. The numerical scheme was initially found to be ill-posed. The application of a Tikhonov regularisation [3] technique made the problem well-posed.
Flow Control

Stabilisation by Continuous Steady Suction

The DVM was modified to simulate suction effects using a 2-D potential sink introduced inside the body, see figure 3. Uniform suction was applied using the spacing between two consecutive wall points \( x_k \) and \( x_{k+1} \) as suction panels. For each panel \((x_k, x_{k+1})\), the suction flow rate \( Q_k \) is obtained as

\[
\psi(x_{k+1}, t) - \psi(x_k, t) = Q_k
\]

where \( \psi \) is a stream function. In order to determine the best location for suction, the numerical simulation was performed with a constant distributed suction at two distinctive locations; one towards the airfoil’s trailing edge and the other along the cavity’s surface. The results for the drag history are shown in figure 5. Averaged numerical values of drag and lift coefficients, \( C_{Dav} \), and \( C_{Lav} \), are also shown. Compared with the result with no control, the placement of the suction system along the cavity was effective in stabilising the trapped vortex by reducing the amplitudes of the unsteady drag force, resulting in a substantial enhancement of the aerodynamic performance. On the contrary, trailing edge suction seems to have a counter productive effect exhibited by the disruption in the flow field even more than in the uncontrolled case, causing higher drag oscillations and reduced lift (from 1.8365 to 1.4469). Figure 6 shows velocity vector plots of the flow with and without suction. The cavity becomes completely filled with an organised vortex bubble when a strong suction is applied compared to the case with no suction.

Force Behaviour with Suction

Figure 7 illustrates the variations in lift and drag forces with increasing suction: the drag decreases and lift increases. This improvement was due to the fact that as suction increases it gradually weakens the vortex shedding process (gradual stabilisation). However, beyond a suction rate of about \( Q = 0.017 \) further increases in suction leads to a decrease in performance.

Stabilisation by Unsteady Suction

The main idea behind this was to cease suction completely once the trapped vortex was stabilised then switch it back on at a later time. This technique relies on the fact that the self rotational behaviour of the trapped vortex enables it to remain stable for some time before its stability is lost. When suction was switched between \( Q_{total} = 0.02 \) and zero the trapped vortex was kept in the cavity with no shedding and a 13% reduction in the required suction was achieved. Interestingly, compared to the case with continuous suction, the \( C_{Lav} \) remained the same but \( C_{Dav} \) was reduced from 0.078 to 0.020 using unsteady suction.

Stability of the Trapped Vortex

The results presented for the Lighthill’s airfoil have clearly shown that with a strong suction a forming massive vortex struc-
ture is withheld permanently in the vicinity of the airfoil and the flow reaches and maintains a steady state. Despite this evidence, we seek a convincing mathematical argument for stability. From the work of Saffman [11], the equilibrium location of a stationary vortex is considered stable if the vortex returns to it after being subjected to a small perturbation. This means that the response of a system to a given perturbation does not develop an instability which destabilises the system, but one which decays with time. Such behaviour is true for a system described by a decaying exponential of the form

$$v(t) = a_0 + a_1 \exp(\lambda t), \quad \lambda < 0.$$  \hspace{1cm} (2)

where $v(t)$ is any flow variable, $a_0$ and $a_1$ are constant coefficients. In order to establish such behaviour for the trapped vortex, it is sufficient to register in time any flow variable like velocity (at a given location) and to see whether it can be represented by a decaying exponential. At a suction rate of 0.02, a sample of registered data for the normal velocity $v$ is fitted with the exponential model as depicted in figure 8. Figure 8 clearly shows that the velocity behaviour asymptotes to a constant value, implying a steady state situation. For suction rates $Q \geq 0.02$, similar behaviour was seen.

**Feedback Control**

**Pre-control Issues**

The presence of a high noise level severely limits the effectiveness of a flow control strategy [8]. Vortex methods are known for being particularly noisy. We used a Tikhonov regularisation technique to achieve some noise reduction in the numerical scheme, see Bouferrouk et al. [2]. The trapped vortex system was found to possess an interesting behaviour depending on the initial conditions. For impulsive starts, the drag values obtained as suction was varied from 0.0 to 0.02 are higher than those obtained when the flow was started from a stable state with $Q = 0.02$ after which suction was slowly reduced to $Q = 0.0$. Figure 9 shows that for the same given suction rate the drag obtained with the slow suction reduction technique is lower than that obtained from the impulsive start. This is not, however, a hysteresis behaviour: when suction is started at $Q = 0.0$ then increased slowly to $Q = 0.02$ and then reduced back to $Q = 0.0$ the drag traces the same path. This could be specific to the current airfoil geometry as results from another Lighthill’s geometry with a different cavity shape (not shown) reveal that the two paths were slightly different, i.e., there is some form of hysteresis. This may potentially be useful in future trapped vortex stabilisation studies.

**The Active Control Law**

The aim is to stabilise the flow and to delay the onset of vortex shedding for as long as possible. Also, it is desired to achieve this with minimum possible suction. The reasons for this are numerous. In a more realistic turbulent flow with a suction system applied at the wall, removal of fluid accelerates the loss of flow momentum with the consequence of increased skin friction drag. Minimising suction also minimises the weight needed to have a suction pump installed. We propose to use a feedback control law, based on an artificial stabilising parameter such that stability is maintained while slowly reducing the suction from a reference stable point. We consider a Single Input Single Output (SISO) standard linear (small perturbation) controller design with a constant gain parameter $G$ (the stabilising parameter) in the form

$$Q_{\text{total}} = Q_0 + G(V_i - V_{\text{bias}})$$  \hspace{1cm} (3)

where $V_i$ is the signal (or sensor variable) taken as the tangential velocity, $V_{\text{bias}}$ is the constant biased velocity (averaged velocity of a flow run without control), $Q_0 = 0.02$, and $Q_{\text{total}}$ is the control variable representing the total suction rate to be varied. This representation implies that the system is only stable for small perturbations and these are modelled in the form of a slow reduction in suction flow rate. Then, by introducing various values of $G$ it is possible to change suction such that the stability of the trapped vortex is maintained for as long as possible before the onset of vortex shedding (the unstable system). The best $G$ would be the one which delays the instability with as little suction as possible.

Figures 10 and 11 illustrate respectively the drag behaviour with two selected values of $G = -0.05$ and $-0.09$, at a fixed monitoring point. It is concluded from the figures that with active control the large perturbations which correspond to large-scale vortex shedding only appear after a more prolonged period of time compared with the uncontrolled case. When large-scale vortex shedding sets in the trapped vortex system is said to have lost its stability. Prior to this, whether control is or is not implemented, some occasional small-scale vortex shedding was observed. Despite this, the main bulk of the trapped vortex flow remains within the cavity and so it is considered stable. Positive values of $G$ were found to be destabilising. However, there seems to be a critical value of $G = -0.09$ beyond which the control actually promotes vortex shedding. A total of 20 sensor positions were investigated in search for an optimum sensor position which delays vortex shedding with the least amount of suction. In terms of suction, the case with $G = -0.09$ consumed more suction compared with $G = -0.05$. The averaged drag coefficient for the case of $G = -0.09$ was 0.0367 while that for $G = -0.05$ was 0.0338.

**Power Balance for the Flow Control**

It is important to evaluate the balance between the power saved by delay of vortex shedding and that consumed by the control flow. This is currently being investigated. Based on the work of Rioual et al. [10] the components of the power balance consist...
of power consumption due to the pressure drag and the power needed to suck fluid through the various panels. The power to account for the momentum loss at the surface of the suction panels may be ignored. The power required to overcome the drag by a propulsive system is given by

\[ P_{\text{drag}} = \frac{D}{\eta_p} \frac{U_{\text{in}}}{\eta_s} \]  

and that required to drive the suction system is expressed as

\[ P_{\text{suc}} = \frac{1}{2\eta_s} \frac{m}{\rho} U_s^2 + L \]  

where \( \eta_p(<1), \eta_s(<1) \) are coefficients reflecting the efficiency of the propulsive and suction systems, respectively, \( m \) the total suction mass flow rate, \( U_s \) the suction velocity, and \( L \) represents energy losses due to friction across the suction surface, and is expressed as (see Bieler et al. [1])

\[ L = \frac{m}{\rho \eta_s} \sum \Delta p_i \]  

where \( \Delta p_i \) is the pressure drop across a suction panel \( i \) and measured as

\[ \Delta p_i = \frac{1}{2} \rho C \left( \frac{U_s}{P} \right)^2 + 32 \mu K \frac{h}{d^2 \frac{U_s}{P}} \]  

In (7), \( C \) and \( K \) are empirical constants, \( d \) is a typical diameter of the hole in the panel, \( h \) is the panel thickness, \( P \) the panel porosity. The values for the various parameters used in (4)-(7) will have to be chosen with suitable assumptions in order to have a reasonable power estimate. This is under investigation.

**Conclusions**

The stabilisation problem of a large-scale vortex structure trapped in a Lighthill’s airfoil cavity has been studied using a DVM. While the cavity entrains the large vortex, continuous increase of suction was required to trap the vortex and inhibit vortex shedding. As a result, the drag decreased and lift increased. However, this is only possible up to a critical suction rate beyond which performance decreases. Because of the rotational character of the trapped vortex, unsteady suction achieved 13% reduction in total suction rate needed for stabilisation. The trapped vortex was shown to be stable using a simple decaying exponential model. By starting at a stable position then slowly reducing the suction rate it is possible to reduce suction further (hence drag) while keeping the vortex stably trapped. Stability is only defined with respect to large-scale vortex shedding of the main trapped vortex as small-scale shedding may occur. The stabilisation was then investigated using a feedback control law based on a constant stabilising parameter. There is evidence that such control retains the vortex stability (with respect to large-scale vortex shedding) for longer periods of time compared with the case of no control. Open questions remain on the optimisation of such control and power balance requirements.

**Acknowledgement**

This research was supported in part by the Royal Society grant (Ref. 13872) and partly by a PhD studentship from the School of Engineering Sciences, University of Southampton.

**References**


